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ON THE SOLVABILITY OF PROBLEMS WITH AN OBLIQUE DERIVATIVE FOR A MIXED PARABOLIC-HYPERBOLIC EQUATION

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Annatation
In this paper, the correctness of the problem with an oblique derivative is proved for a mixed parabolic-hyperbolic equation with a characteristic line of type change.

Аннатация
В настоящей работе доказанно корректность задачи с наклонной производной, для смешанного параболо-гиперболического уравнения с характеристической линией изменения типа.

Keywords: parabolic-hyperbolic equations, Hausdorff space, Banach space

Introduction

The main goal of this paper is to show that the well-posedness of the problem with an oblique derivative given at the departure from the characteristic for a mixed parabolic-hyperbolic equation with a characteristic line of type change depends significantly on the class in which the solution is sought and on the value of the polar angle formed by the vector the field defining the oblique derivative, and the abscissa axis at the beginning of the departure.

Let $\Omega \subset R^2$ - be a finite region bounded for y>0 by segments AA_0, A_0B_0, BB_0, A(0,0), $A_0(0,1)$, B(1,0), $B_0(1,1)$, and for y<0 - a smooth curve , $AD: y = -\gamma(x), 0 \le x \le \ell$, where $0{,}5 < \ell \le 1,\ \gamma(0) = 0,\ \ell + \gamma(\ell) = 1$, located inside the characteristic triangle $ABC: 0 \le x + y < x - y \le 1$, and a segment of the characteristic $BC: x - y = 1,\ \ell \le x \le 1$, equations of mixed parabolic-hyperbolic type

$$Lu = f(x, y), \qquad (1)$$

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Where

$$Lu = \begin{cases} u_{xx} - u_y, & y > 0 \\ u_{xx} - u_{yy}, & y < 0. \end{cases}$$

With respect to the curve AD, we will assume that at none of its points it has a characteristic direction, that it belongs to the class of functions $C^1[0,\ell]$ and $\gamma(0)=0$, $0 < x \le \ell < 1, \quad \ell + \gamma(\ell) = 1,$, that the function $x+\gamma(x)$ is monotonically increasing.

Task BM. Find a solution to equation (1) that satisfies the conditions

$$u\big|_{AA_0 \cup BB_0} = 0, \qquad (2)$$

$$\left[a(x,y)u_x + b(x,y)u_y + c(x,y)u\right]_{AD} = 0, \qquad (3)$$

where $a(x,y), b(x,y), c(x,y)$ - given functions.

For domains D such that $\overline{D} \cap \{A\} \neq \emptyset$, we define a function space

$$C_\alpha^{m.n}(\overline{D}) = \Big\{ u(x,y) : \frac{\partial^{i+j} u(x,y)}{\partial x^i \partial y^j} \in C(\overline{D}), \quad \frac{\partial^{i+j} u(0,0)}{\partial x^i \partial y^j} = 0, \ \ i + \frac{m}{n} j \le m-1, \ \ m \ge n \ge 0,$$

$$\frac{\partial^{i+j} u(x,y)}{\partial x^i \partial y^j} \in C(\overline{D} \setminus A), \quad i + \frac{m}{n} j = m, \qquad \max_{i+\frac{m}{n}j=m} \ \sup_{x \in \overline{D}} \left| x^{-\alpha} \frac{\partial^{i+j} u(x,y)}{\partial x^i \partial y^j} \right| < \infty, \alpha \ge 0 \Big\}.$$

It is clear that with respect to the norm

$$\|u\|_{m,n,\alpha} = \max_{i+\frac{m}{n}j=m} \ \sup_{(x,y)\in \overline{D}} \left| x^{-\alpha} \frac{\partial^{i+j} u(x,y)}{\partial x^i \partial y^j} \right|$$

the $C_\alpha^{m.n}(\overline{D})$ space is Banach. Let $a(x,y) \neq b(x,y)$. We denote θ by the polar angle formed by the vector field that defines the oblique derivative, and by the abscissa axis at the beginning of the departure (at point A).

Theorem.1. If $\frac{\pi}{2} < \theta < 0$ or $\frac{\pi}{2} < \theta < \pi$, then for any function

$$f(x,y) \in C_\alpha^{0.0}(\overline{\Omega}_0) \cap C_\alpha^{1.1}(\overline{\Omega}_1) \cap C^{1.0}(\Omega_0)$$

there is a unique regular solution of the BM problem from the class

$$u(x,y) \in C^{1.1}(\overline{\Omega} \setminus B) \cap C_\alpha^{2.1}(\overline{\Omega}_0 \setminus B) \cap C_\alpha^{2.2}(\overline{\Omega}_1).$$

If $0 < \theta < \frac{\pi}{2}$ or $\pi < \theta < \frac{3\pi}{2}$, then the BM problem in the specified class inormally solvable in the sense of Hausdorff and its index $\chi = +\infty$ and, in particular, the corresponding homogeneous problem has an infinite set of linearly independent solutions.

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