

SOME INTEGRAL EQUATIONS FOR A MULTIVARIABLE FUNCTION

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Annotation

In this article some integral equations for a multivariable function are learned and solution is found using by replacing, multiplying and integrating.

Keywords: integral equations, multivariable function, differentiate and substituting.

Introduction

The equation in the form of this

$$\frac{1}{\Gamma(\alpha_1)\Gamma(\alpha_2)} \int_{a_{x_2}}^{x_1} \int_{a_{x_2}}^{b} (x_1 - t_1)^{\alpha_1 - 1} (t_2 - x_2)^{\alpha_2 - 1} \varphi(t_1, t_2) dt_1 dt_2 = f(x_1, x_2)$$
(1)

is an integral equation on the arguments x_1 and x_2 from the function $\varphi(x_1, x_2)$ of two variables. Here a, b, α_1 and α_2 is real numbers, $0 < \alpha_1, \alpha_2 < 1$.

The above integral equation can be written as

$$\frac{1}{\Gamma(\alpha_1)\Gamma(\alpha_2)} \int_{a}^{x_1} (x_1 - t_1)^{\alpha_1 - 1} dt_1 \int_{x_2}^{b} (t_2 - x_2)^{\alpha_2 - 1} \varphi(t_1, t_2) dt_2 = f(x_1, x_2).$$
(2)

In this expression we enter the designation is

$$\int_{x_2}^{b} (t_2 - x_2)^{\alpha_2 - 1} \varphi(t_1, t_2) dt_2 = \psi_1(t_1, x_2).$$
(3)

In this case, the equation (2) can be written as

$$\frac{1}{\Gamma(\alpha_1)\Gamma(\alpha_2)} \int_{a}^{x_1} \frac{\psi_1(t_1, x_2) dt_1}{(x_1 - t_1)^{1 - \alpha_1}} = f(x_1, x_2).$$
(4)

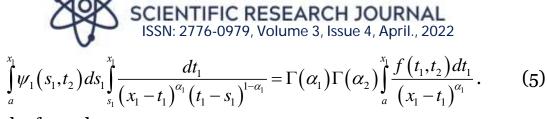
In equation (4) like [1] work by replacing x_1 with t_1 and t_1 with s_1 , then we multiply both sides of the equation by the expression $(x_1 - t_1)^{-\alpha_1}$ and we integrate from a to x_1 on t_1

$$\frac{1}{\Gamma(\alpha_1)\Gamma(\alpha_2)} \int_a^{x_1} \frac{dt_1}{(x_1-t_1)^{\alpha_1}} \int_a^{t_1} \frac{\psi_1(s_1,t_2)ds_1}{(t_1-s_1)^{1-\alpha_1}} = \int_a^{x_1} \frac{f(t_1,t_2)dt_1}{(x_1-t_1)^{\alpha_1}}.$$

In this expression we replace the order of integration according to the Direxli formula as in the case [2]



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If we use the formula

$$B(a,b) = \int_{0}^{1} x^{a-1} (1-x)^{b-1} dx$$
(6)

by substituting $t_1 = s_1 + (x_1 - s_1)z_1$ in the integral to the left of this equation, we get the equation

$$\int_{a}^{x_{1}} (x_{1}-t_{1})^{-\alpha_{1}} (t_{1}-s_{1})^{\alpha_{1}-1} dt_{1} = \Gamma(\alpha_{1})\Gamma(1-\alpha_{1}).$$

In this case, (5) is based on equation

$$\int_{a}^{x_{1}} \psi_{1}(s_{1},t_{2}) ds_{1} = \frac{\Gamma(\alpha_{2})}{\Gamma(1-\alpha_{1})} \int_{a}^{x_{1}} \frac{f(t_{1},t_{2}) dt_{1}}{(x_{1}-t_{1})^{\alpha_{1}}}$$

We differentiate both sides of this equation by x_1 . As the result, this

$$\Psi(x_1, x_2) = \frac{\Gamma(\alpha_2)}{\Gamma(1-\alpha_1)} \frac{d}{dx_1} \int_a^{x_1} \frac{f(t_1, t_2) dt_1}{(x_1 - t_1)^{\alpha_1}}$$

equality can be created. In this expression by replacing x_1 with t_1 and t_1 with s_1 , we put in (3) equation

$$\int_{x_{2}}^{b} \frac{\varphi(t_{1},t_{2})dt_{2}}{(t_{2}-x_{2})^{1-\alpha_{2}}} = \frac{\Gamma(\alpha_{2})}{\Gamma(1-\alpha_{1})} \frac{d}{dt_{1}} \int_{a}^{t_{1}} \frac{f(s_{1},x_{2})ds_{1}}{(t_{1}-s_{1})^{\alpha_{1}}} .$$
(7)

In equation (7) like [1] work by replacing x_2 with t_2 and t_2 with s_2 , then we multiply both sides of the equation by the expression $(t_2 - x_2)^{-\alpha_2}$ and we integrate x_2 to b on t_2

$$\int_{x_{2}}^{b} \frac{dt_{2}}{(t_{2}-x_{2})^{-\alpha_{2}}} \int_{t_{2}}^{b} \frac{\varphi(t_{1},s_{2})ds_{2}}{(s_{2}-t_{2})^{1-\alpha_{2}}} = \frac{\Gamma(\alpha_{2})}{\Gamma(1-\alpha_{1})} \int_{x_{2}}^{b} \frac{dt_{2}}{(t_{2}-x_{2})^{\alpha_{2}}} \frac{d}{dt_{1}} \int_{a}^{t_{1}} \frac{f(s_{1},x_{2})ds_{1}}{(t_{1}-s_{1})^{\alpha_{1}}}$$

In this expression we replace the order of integration according to the Direxli formula as in the case [2]

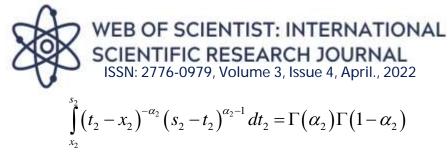
$$\int_{x_{2}}^{b} \varphi(t_{1},s_{2}) ds_{2} \int_{x_{2}}^{s_{2}} \frac{dt_{2}}{(t_{2}-x_{2})^{\alpha_{2}} (s_{2}-t_{2})^{1-\alpha_{2}}} = \frac{\Gamma(\alpha_{2})}{\Gamma(1-\alpha_{1})} \int_{x_{2}}^{b} \frac{dt_{2}}{(t_{2}-x_{2})^{\alpha_{2}}} \frac{d}{dt_{1}} \int_{a}^{t_{1}} \frac{f(s_{1},x_{2}) ds_{1}}{(t_{1}-s_{1})^{\alpha_{1}}}.$$
 (8)

If we use the formula (6) by substituting $t_2 = x_2 + (s_2 - x_2)z_2$ in the internal integral on the left side of the equation,



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equation is formed.

Then we have

$$\int_{x_{2}}^{b} \varphi(t_{1},s_{2}) ds_{2} = \frac{1}{\Gamma(1-\alpha_{1})\Gamma(1-\alpha_{2})} \int_{x_{2}}^{b} \frac{dt_{2}}{(t_{2}-x_{2})^{\alpha_{2}}} \frac{d}{dt_{1}} \int_{a}^{t_{1}} \frac{f(s_{1},x_{2}) ds_{1}}{(t_{1}-s_{1})^{\alpha_{1}}}$$

expression based on (8).

We differentiate both sides of this equation by x_2 and we form the solution of this equation (1):

$$\varphi(x_1, x_2) = -\frac{1}{\Gamma(1 - \alpha_1)\Gamma(1 - \alpha_2)} \frac{d^2}{dx_1 dx_2} \int_{a}^{x_1} \int_{a}^{b} \frac{f(t_1, t_2) dt_1 dt_2}{(x_1 - t_1)^{\alpha_1} (t_2 - x_2)^{\alpha_2}} .$$

Like this the solution of this

$$\frac{1}{\Gamma(\alpha_{1})...\Gamma(\alpha_{k})\Gamma(\alpha_{k+1})...\Gamma(\alpha_{n})} \int_{a}^{x_{1}} ... \int_{a}^{x_{k}} \int_{x_{k+1}}^{b} ... \int_{x_{n}}^{b} (x_{1} - t_{1})^{\alpha_{1} - 1} ... (x_{k} - t_{k})^{\alpha_{k} - 1} \times (t_{k+1} - x_{k+1})^{\alpha_{k+1} - 1} ... (t_{n} - x_{n})^{\alpha_{n} - 1} \varphi(t_{1}, ..., t_{k}, t_{k+1}, ..., t_{n}) \times dt_{1} ... dt_{k} dt_{k+1} ... dt_{n} = f(x_{1}, ..., x_{k}, x_{k+1}, ..., x_{n})$$

equation, can be proved like that

$$\varphi(x_{1},...,x_{k},x_{k+1},...,x_{n}) = \frac{(-1)^{n-k}}{\Gamma(1-\alpha_{1})...\Gamma(1-\alpha_{k})\Gamma(1-\alpha_{k+1})...\Gamma(1-\alpha_{n})} \times \frac{d^{n}}{dx_{1}...dx_{k}dx_{k+1}...dx_{n}} \int_{a}^{x_{1}}...\int_{a}^{x_{k}} \int_{x_{n+1}}^{b}...\int_{x_{n}}^{b} \frac{f(t_{1},...,t_{k},t_{k+1},...,t_{n})dt_{1}...dt_{k}dt_{k+1}...dt_{n}}{(x_{1}-t_{1})^{\alpha_{1}}...(x_{k}-t_{k})^{\alpha_{k}}(t_{k+1}-x_{k+1})^{\alpha_{k+1}}...(t_{n}-x_{n})^{\alpha_{n}}}$$

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