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SOME INTEGRAL EQUATIONS FOR A MULTIVARIABLE FUNCTION

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Annotation
In this article some integral equations for a multivariable function are learned and solution is found using by replacing, multiplying and integrating.

Keywords: integral equations, multivariable function, differentiate and substituting.

Introduction
The equation in the form of this

$$\frac{1}{\Gamma(\alpha_1)\Gamma(\alpha_2)}\int_a^{x_1}\int_{x_2}^{b}(x_1-t_1)^{\alpha_1-1}(t_2-x_2)^{\alpha_2-1}\varphi(t_1,t_2)dt_1dt_2 = f(x_1,x_2) \quad (1)$$

is an integral equation on the arguments x_1 and x_2 from the function $\varphi(x_1,x_2)$ of two variables. Here a,b,α_1 and α_2 is real numbers, $0<\alpha_1,\alpha_2<1$.
The above integral equation can be written as

$$\frac{1}{\Gamma(\alpha_1)\Gamma(\alpha_2)}\int_a^{x_1}(x_1-t_1)^{\alpha_1-1}dt_1\int_{x_2}^{b}(t_2-x_2)^{\alpha_2-1}\varphi(t_1,t_2)dt_2 = f(x_1,x_2). \quad (2)$$

In this expression we enter the designation is

$$\int_{x_2}^{b}(t_2-x_2)^{\alpha_2-1}\varphi(t_1,t_2)dt_2 = \psi_1(t_1,x_2). \quad (3)$$

In this case, the equation (2) can be written as

$$\frac{1}{\Gamma(\alpha_1)\Gamma(\alpha_2)}\int_a^{x_1}\frac{\psi_1(t_1,x_2)dt_1}{(x_1-t_1)^{1-\alpha_1}} = f(x_1,x_2). \quad (4)$$

In equation (4) like [1] work by replacing x_1 with t_1 and t_1 with s_1, then we multiply both sides of the equation by the expression $(x_1-t_1)^{-\alpha_1}$ and we integrate from a to x_1 on t_1

$$\frac{1}{\Gamma(\alpha_1)\Gamma(\alpha_2)}\int_a^{x_1}\frac{dt_1}{(x_1-t_1)^{\alpha_1}}\int_a^{t_1}\frac{\psi_1(s_1,t_2)ds_1}{(t_1-s_1)^{1-\alpha_1}} = \int_a^{x_1}\frac{f(t_1,t_2)dt_1}{(x_1-t_1)^{\alpha_1}}.$$

In this expression we replace the order of integration according to the Direxli formula as in the case [2]

$$\int_a^{x_1}\psi_1(s_1,t_2)ds_1\int_{s_1}^{x_1}\frac{dt_1}{(x_1-t_1)^{\alpha_1}(t_1-s_1)^{1-\alpha_1}}=\Gamma(\alpha_1)\Gamma(\alpha_2)\int_a^{x_1}\frac{f(t_1,t_2)dt_1}{(x_1-t_1)^{\alpha_1}}. \quad (5)$$

If we use the formula

$$B(a,b)=\int_0^1 x^{a-1}(1-x)^{b-1}dx \quad (6)$$

by substituting $t_1=s_1+(x_1-s_1)z_1$ in the integral to the left of this equation, we get the equation

$$\int_a^{x_1}(x_1-t_1)^{-\alpha_1}(t_1-s_1)^{\alpha_1-1}dt_1=\Gamma(\alpha_1)\Gamma(1-\alpha_1).$$

In this case, (5) is based on equation

$$\int_a^{x_1}\psi_1(s_1,t_2)ds_1=\frac{\Gamma(\alpha_2)}{\Gamma(1-\alpha_1)}\int_a^{x_1}\frac{f(t_1,t_2)dt_1}{(x_1-t_1)^{\alpha_1}}.$$

We differentiate both sides of this equation by x_1.
As the result, this

$$\psi(x_1,x_2)=\frac{\Gamma(\alpha_2)}{\Gamma(1-\alpha_1)}\frac{d}{dx_1}\int_a^{x_1}\frac{f(t_1,t_2)dt_1}{(x_1-t_1)^{\alpha_1}}$$

equality can be created. In this expression by replacing x_1 with t_1 and t_1 with s_1, we put in (3) equation

$$\int_{x_2}^{b}\frac{\varphi(t_1,t_2)dt_2}{(t_2-x_2)^{1-\alpha_2}}=\frac{\Gamma(\alpha_2)}{\Gamma(1-\alpha_1)}\frac{d}{dt_1}\int_a^{t_1}\frac{f(s_1,x_2)ds_1}{(t_1-s_1)^{\alpha_1}}. \quad (7)$$

In equation (7) like $[1]$ work by replacing x_2 with t_2 and t_2 with s_2, then we multiply both sides of the equation by the expression $(t_2-x_2)^{-\alpha_2}$ and we integrate x_2 to b on t_2

$$\int_{x_2}^{b}\frac{dt_2}{(t_2-x_2)^{-\alpha_2}}\int_{t_2}^{b}\frac{\varphi(t_1,s_2)ds_2}{(s_2-t_2)^{1-\alpha_2}}=\frac{\Gamma(\alpha_2)}{\Gamma(1-\alpha_1)}\int_{x_2}^{b}\frac{dt_2}{(t_2-x_2)^{\alpha_2}}\frac{d}{dt_1}\int_a^{t_1}\frac{f(s_1,x_2)ds_1}{(t_1-s_1)^{\alpha_1}}.$$

In this expression we replace the order of integration according to the Direxli formula as in the case $[2]$

$$\int_{x_2}^{b}\varphi(t_1,s_2)ds_2\int_{x_2}^{s_2}\frac{dt_2}{(t_2-x_2)^{\alpha_2}(s_2-t_2)^{1-\alpha_2}}=\frac{\Gamma(\alpha_2)}{\Gamma(1-\alpha_1)}\int_{x_2}^{b}\frac{dt_2}{(t_2-x_2)^{\alpha_2}}\frac{d}{dt_1}\int_a^{t_1}\frac{f(s_1,x_2)ds_1}{(t_1-s_1)^{\alpha_1}}. \quad (8)$$

If we use the formula (6) by substituting $t_2=x_2+(s_2-x_2)z_2$ in the internal integral on the left side of the equation,

$$\int_{x_2}^{s_2}(t_2-x_2)^{-\alpha_2}(s_2-t_2)^{\alpha_2-1}dt_2=\Gamma(\alpha_2)\Gamma(1-\alpha_2)$$

equation is formed.

Then we have

$$\int_{x_2}^{b}\varphi(t_1,s_2)ds_2=\frac{1}{\Gamma(1-\alpha_1)\Gamma(1-\alpha_2)}\int_{x_2}^{b}\frac{dt_2}{(t_2-x_2)^{\alpha_2}}\frac{d}{dt_1}\int_{a}^{t_1}\frac{f(s_1,x_2)ds_1}{(t_1-s_1)^{\alpha_1}}$$

expression based on (8).

We differentiate both sides of this equation by x_2 and we form the solution of this equation (1):

$$\varphi(x_1,x_2)=-\frac{1}{\Gamma(1-\alpha_1)\Gamma(1-\alpha_2)}\frac{d^2}{dx_1dx_2}\int_{a}^{x_1}\int_{x_2}^{b}\frac{f(t_1,t_2)dt_1dt_2}{(x_1-t_1)^{\alpha_1}(t_2-x_2)^{\alpha_2}}\quad .$$

Like this the solution of this

$$\frac{1}{\Gamma(\alpha_1)\ldots\Gamma(\alpha_k)\Gamma(\alpha_{k+1})\ldots\Gamma(\alpha_n)}\int_{a}^{x_1}\ldots\int_{a}^{x_k}\int_{x_{k+1}}^{b}\ldots\int_{x_n}^{b}(x_1-t_1)^{\alpha_1-1}\ldots(x_k-t_k)^{\alpha_k-1}\times$$
$$\times(t_{k+1}-x_{k+1})^{\alpha_{k+1}-1}\ldots(t_n-x_n)^{\alpha_n-1}\varphi(t_1,\ldots,t_k,t_{k+1},\ldots,t_n)\times$$
$$\times dt_1\ldots dt_k dt_{k+1}\ldots dt_n=f(x_1,\ldots,x_k,x_{k+1},\ldots,x_n)$$

equation, can be proved like that

$$\varphi(x_1,\ldots,x_k,x_{k+1},\ldots,x_n)=\frac{(-1)^{n-k}}{\Gamma(1-\alpha_1)\ldots\Gamma(1-\alpha_k)\Gamma(1-\alpha_{k+1})\ldots\Gamma(1-\alpha_n)}\times$$
$$\times\frac{d^n}{dx_1\ldots dx_k dx_{k+1}\ldots dx_n}\int_{a}^{x_1}\ldots\int_{a}^{x_k}\int_{x_{k+1}}^{b}\ldots\int_{x_n}^{b}\frac{f(t_1,\ldots,t_k,t_{k+1},\ldots,t_n)dt_1\ldots dt_k dt_{k+1}\ldots dt_n}{(x_1-t_1)^{\alpha_1}\ldots(x_k-t_k)^{\alpha_k}(t_{k+1}-x_{k+1})^{\alpha_{k+1}}\ldots(t_n-x_n)^{\alpha_n}}.$$

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