WEB OF SCIENTIST: INTERNATIONAL SCIENTIFIC RESEARCH JOURNAL
ISSN: 2776-0979, Volume 3, Issue 4, April., 2022

NONLOCAL BOUNDARY VALUE PROBLEMS FOR A THIRD-ORDER MIXED-COMPOSITE TYPE EQUATION

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Annotation

The study of boundary value problems for an equation of composite type is a relatively new direction in the theory of boundary value problems. These problems are of particular interest in connection with their application in various problems of mechanics and physics, such they arise when modeling heat and mass transfer in capillary-porous media of a number of different biological objects and other problems. The present work is devoted to the study of boundary value problems for a third-order equation of a shifted composite type.

Keywords: Nonlocal, nonlocal problem, equations of mixed-composite type, locale, singular integral equation.

Introduction

In a simply connected region D bounded by a smooth line σ, based on dots $A(0;1)$ and $B(1;0)$ located in a quarter plane $(x>0,\ y<0)$ and segments $AA_1,\ BE,\ A_1E$ directly $x=0,\ x=1,\ y=1$ respectively, where $O,\ E-$ points with coordinates $(0,0),\ (1,1)$ the equations

$$\frac{\partial}{\partial x}(Lu)=0, \tag{1}$$

where $Lu=u_{xx}+\dfrac{1-\operatorname{sgn} y}{2}u_{yy}-\dfrac{1+\operatorname{sgn} y}{2}u_y$.

Task 1. Find a function $u(x,y)$ with the following properties:

1. Function $u(x,y)$ is a regular solution of Eq. (1) in the domain $D\,(y\neq 0)$.
2. Function $u(x,y)$ and its partial derivatives of the first order are continuous in a closed region (it is assumed that at the points $O(0,0)$, $B(1,0)$ partial derivatives u_x, u_y can go to infinity of order less than one).
3. Function $u(x,y)$ satisfies the boundary conditions.

$$u\big|_{\sigma} = f(\xi),\, u\big|_{BE} = \psi_1(y),\, u(0,y)+u(0,-y)\big|_{AA_1} = \psi(y),$$
$$u_x\big|_{AA_1} = \nu(y),\, \frac{\partial u}{\partial n}\big|_{\sigma} = f_1(\xi), \qquad (2)$$

where f, $f_1, \psi, \psi_1, \nu -$ given functions that satisfy certain smoothness conditions and matching conditions, and $\psi(y)-$ smooth function.

Task 2. Find a function $u(x,y)$ with the properties of problem 1 except for the boundary condition $u\big|_{BE} = \varphi_1(y)$, which is replaced by the condition $u\big|_{x=1} = u\big|_{BE}$ $(0<e<1)$.

When studying these problems, we will use the factor that any regular solution to Eq. (1) can be represented in the form

$$u(x,y) = z(x,y) + \mu(y), \qquad (3)$$

respectively [1-6], where $z(x,y)-$ regular solution of the equation

$$u_{xx} + \frac{1+\operatorname{sgn} y}{2} u_{yy} - \frac{1+\operatorname{sgn} y}{2} u_y = 0. \qquad (4)$$

Denoting $\mu(y) = \begin{cases} \mu_1(y), & 0\le y\le 1, \\ \mu_2(y), & -1\le y\le 0, \end{cases}$ $\mu_2(x,y)-$ arbitrary doubly continuously differentiable function, $\mu_1(y)-$ arbitrary continuously differentiable function.

Without loss of generality, we can assume $\omega(0)=\omega'(0)=0$. It is assumed that it lies entirely in the strip bounded by the straight line $x=0,\ x=1$.

Without loss of generality, we can assume that $\mu(0)=\mu'(0)=0$.

Based on (3) and (2), problem 2 is reduced to determining a regular solution (4) in the D domain satisfying the boundary conditions

$$z\big|_{\sigma} = f(\xi) - \mu_2(y),\, z\big|_{BE} = \psi_1(y) - \mu_1(y),$$
$$z\big|_{OA_1} = \varphi(y) - \mu_1(y),\, z\big|_{OA} = \psi(y) - \varphi(y) - \mu_2(y), \qquad (5)$$
$$z_x\big|_{AA_1} = \nu(y),\, \frac{\partial z}{\partial \eta}\big|_{\sigma} = f_1(\xi) - \mu_1'(y)\frac{\partial y}{\partial n}.$$

The uniqueness of the solution to Problem 2 follows from the extremum principle. (It is assumed that $\frac{\partial x}{\partial n} \neq 0$ along the arc σ).

Let's define $\nu_1(x)$. Delivering value $\nu_1(x)$ into the formula

$$z(x,y) = \int_0^1 \nu_1(t)G(x,y;t,0)dt + \int_{3\pi/2}^{2\pi} \overline{f}(\theta)\frac{\partial G}{\partial n}\Big|_{|\xi|=1} d\theta - \int_{-1}^{0} \nu(t)G(x,y;0,t)dt$$

have

$$z(x,y) = \int_{3\pi/2}^{2\pi} \mu_2(\sin\theta)\rho_1(x,y;\theta)d\theta + P(x,y), \tag{6}$$

$\rho_1(x,y;\theta)$, $P(x,y)$ – known functions realizing the last condition from (5), to determine $\mu_2'(y)$ we obtain the singular integral equation (3).

$$\begin{aligned}
&\delta(\theta_0)\sin\theta_0 + \frac{\cos\theta_0}{2\pi}\int_{3\pi/2}^{2\pi}\frac{\delta(\theta)}{\theta-\theta_0}d\theta - \frac{\sin\theta_0}{2\pi}\int_{3\pi/2}^{2\pi}\frac{K_1(\theta,\theta_0)-K_1(\theta_0,\theta_0)}{\theta-\theta_0}\delta(\theta)d\theta + \\
&+\frac{\cos\theta_0}{2\pi}\int_{3\pi/2}^{2\pi}\frac{K_2(\theta,\theta_0)-K_2(\theta_0,\theta_0)}{\theta-\theta_0}\delta(\theta)d\theta + \frac{\cos\theta_0}{2\pi}\int_{3\pi/2}^{2\pi} Q(\theta,\theta_0)\delta(\theta)\cos\theta d\theta + \\
&+\frac{\cos\theta_0}{2\pi}\int_{3\pi/2}^{2\pi} Q_1(\theta,\theta_0)\delta(\theta)\cos\theta d\theta = \rho_2(\theta_0),
\end{aligned} \tag{7}$$

Where $\delta(\theta) = \mu_2'(\sin\theta)$; $K_1(\theta,\theta_0)$, $K_2(\theta,\theta_0)$, $Q(\theta,\theta_0)$, $Q_1(\theta,\theta_0)$ – known features.

From equation (7) we define the function $\mu_2(y)$. So the function $u(x,y)$ completely determined in the region D_2.

Solution of equation (4) satisfying the boundary conditions

$$z\big|_{BE} = \psi(y) - \mu_1(y),\ z\big|_{OB} = \tau(x),\ z_x\big|_{A_1B} = \nu(y)$$

is determined by the formula

$$\begin{aligned}
z(x,y) = &\int_0^y \nu(t)\overline{G}(x,y;0,t)dt - \int_0^y [\psi_1(t) - \mu_1(t)]\overline{G}_\xi(x,y;1,t)dt - \\
&-\int_0^y \nu(t)\overline{G}(x,y;t,0)dt,
\end{aligned} \tag{8}$$

where $\overline{G}(x,y;\xi,\eta)$ – Green's function.

Function $z(x,y)$, defined by formula (2) must satisfy the condition $z\big|_{OA_1} = \varphi(y) - \mu_1(y)$,

WEB OF SCIENTIST: INTERNATIONAL SCIENTIFIC RESEARCH JOURNAL
ISSN: 2776-0979, Volume 3, Issue 4, April., 2022

$$\varphi(y)-\mu_1(y)=\int_0^y \nu(t)\overline{G}(0,y;0,t)dt-\int_0^y \tau(t)\overline{G}(0,y;t,0)dt-$$
$$-\int_0^y [\psi(t)-\mu_1(t)]\overline{G}_\xi(0,y;1,t)dt. \tag{9}$$

Realizing the condition $z\big|_{OA_1}=\psi(y)-\varphi(-y)-\mu_2(y)$ have

$$\psi(y)-\varphi(y)-\mu_2(y)=\int \mu_2(\sin\theta)\rho_1(0,y,\theta)d\theta+P(0,y). \tag{10}$$

Delivering value $\varphi(y)$ into formula (9) to determine $\mu_1(y)$, we obtain the Volterra integral equation of the second kind

$$\mu_1(y)+\int_0^y K(y,t)\mu_1(t)dt=P_2(y), \tag{11}$$

where $K(y,t),\ P_2(y)$ – known features.

Equation (11) uniquely determines the function $\mu_1(y)$.

Problem 2 is reduced to finding a regular solution to equation (5) satisfying the boundary conditions

$$\left.\begin{aligned} &z\big|_\sigma=f(\xi)-\mu_2(y),\ z\big|_{x=e}=z\big|_{BE}=\varphi_1(y)-\mu_1(y),\\ &z\big|_{OA_1}=\varphi(y)-\mu_1(y),\ z\big|_{OA}=\psi(y)-\varphi(y)-\mu_2(y),\\ &z_x\big|_{AA_1}=\nu(y)\frac{\partial z}{\partial n}\big|_\sigma=f_1(\xi)-\mu_2^{'}(y)\frac{\partial z}{\partial n}. \end{aligned}\right\} \tag{12}$$

Through $\varphi(y),\ \varphi_1(y)$ values are marked accordingly $u(0,y),\ u(1,y)\ (0\le y\le 1)$.

The uniqueness of the solution to problem 2 is proved using the maximum principle under the assumption that $\frac{\partial x}{\partial n}\neq 0$ along the line.

The existence of a solution to problem 2 of the region D_2 is defined as in the case of problem 1, the solution to equation (5) satisfying the boundary conditions

$$z\big|_{BE}=\varphi_1(y)-\mu_1(y),\ z\big|_{OB}=\tau(x),\ z_x\big|_{OA_1}=\nu(x),$$

is given by the formula

$$z(x,y)=\int_0^y \nu(t)\overline{G}(x,y;0,t)dt-\int_0^1 \tau(t)\overline{G}(x,y;t,0)dt-$$
$$-\int_0^y [\varphi_1(t)-\mu_1(t)]\overline{G}_\xi(x,y;1,t)dt, \tag{13}$$

$z(x,y)$ must satisfy the condition

$$z\big|_{z=e}=\varphi_1(y)-\mu_1(y),$$

$$\varphi_1(y)-\mu_1(y)+\int_0^y[\varphi_1(t)-\mu_1(t)]\overline{G}_\xi(e,y;1,t)dt= \\ =\int_0^y\nu(t)\overline{G}(e,y;0,t)dt-\int_0^1\tau(t)\overline{G}(e,y;t,0)dt. \qquad (14)$$

Integral equation (14) is a Volterra integral equation of the second kind, which uniquely determines $\varphi_1(y)-\mu_1(y)$.

Realizing the condition $z\big|_{OA_1}=\varphi(y)-\mu_1(y)$ from (13) find

$$\mu_1(y)=\varphi(y)+\int_0^y[\varphi_1(t)-\mu_1(t)]\overline{G}_\xi(e,y;1,t)dt+ \\ +\int_0^1\tau(t)G(e,y;t,0)dt+\int_0^1\nu(t)\overline{G}(e,y;0,t)dt. \qquad (15)$$

Delivering value $\varphi(y), \varphi_1(y)-\mu_1(y)$ into the formula (15) define the function $\mu_1(y)$, где $\varphi(y)$ is found from the formula (8).

References

1. Врагов В.И. Об одном уравнении смешанного – составного типа диф. урав. 1973. Т IX, №1. Стр. 169-171.
2. Михлин С.Г. Интегральные уравнения. ОГИЗ. Гостехиздат. 1949.
3. Муминов Ф.М., Душатов Н.Т. Нелокальная краевая задача для линейных уравнений смешанного типа. Central Asian Journal of Theoretical and Applied Sciences, 2021. Vol.02, Issue:05, стр.191-196.
4. Муминов Ф.М., Миратоев З.М. О нелокальной краевой задаче для одного неклассического уравнения. «Scientific progress» Scientific Journal. April 2021. ISSN: 2181-1601 Volume: 1, ISSUE: 6.
5. FM Muminov, NT Dushatov, ZM Miratoev. Boundary Problem for a Third Order Equation of a Mixed Composite Type. Eurasian Journal of Physics, Chemistry and Mathematics. 2022/4/8. Vol.05, ISSN:2795-7667, pp.12-16.
6. Ф М Муминов, З М Миратоев, У А Утабов. Об Одной Краевой Задаче Для Уровнениясоставного Типа Третьего Порядка. CENTRAL ASIAN JOURNAL OF THEORETICAL & APPLIED SCIENCES. Vol.:2. №4 стр. 17-22.
7. Муминов Ф.М., Миратоев З.М., Утабов У. Об одной краевой задаче для уровнения составного типа третьего порядка. CENTRAL ASIAN JOURNAL OF THEORETICAL & APPLIED SCIENCES, 2(4), pp.17-22. 2021.

8. Муминов Ф.М., Миратоев З.М. О нелокальной краевой задаче для одного неклассического уравнения. «Scientificprogress» ScientificJournal. April 2021. ISSN: 2181-1601 Volume: 1, ISSUE: 6.
9. Muminov F.M, Dushatov N.T, & Miratoev Z.M. (2022). Boundary Problem for a Third Order Equation of a Mixed Composite Type. *Eurasian Journal of Physics,Chemistry and Mathematics*, *5*, 12–16.
10. Муминов, Ф.М., Самандаров, И.Р., Душатов, Н.Т., Миратоев, З.М. О НЕКОТОРЫХ ЗАДАЧАХ ДЛЯ УРАВНЕНИЯ ТРЕТЬЕГО ПОРЯДКА СМЕЩАННОГО-СОСТАВНОГО ТИПА // ORIENSS. 2022. №4. URL: https://cyberleninka.ru/article/n/o-nekotoryh-zadachah-dlya-uravneniya-tretiego-poryadka-smeschannogo-sostavnogo-tipa (дата обращения: 17.04.2022).
11. Муминов, Ф.М., Самандаров, И.Р., Душатов, Н.Т., Миратоев, З.М. КРАЕВАЯ ЗАДАЧА ДЛЯ ОДНОГО КЛАССА УРАВНЕНИЙ ТРЕТЬЕГО ПОРЯДКА // ORIENSS. 2022. №4. URL: https://cyberleninka.ru/article/n/kraevaya-zadacha-dlya-odnogo-klassa-uravneniy-tretiego-poryadka (дата обращения: 17.04.2022).
12. Abdushukurov A. A., Muradov R. S. On estimation of conditional distribution function under dependent random right censored data. – 2014.
13. Abdushukurov A. A., Dushatov N. T., Muradov R. S. Estimating of functionals of a multidimensional distribution by censored observations with using copula functions //Statistical Methods of Estimation and Hyphoteses Testing. Perm State University. – 2011. – №. 23. – С. 36-47.
14. Dushatov N. T., Muradov R. S. Estimation of multivariate survival function by dependent censored data //Applied Methods of Statistical Analysis. Simulations and Statistical Inference. Book of Abstracts of the International Workshop. Novosibirsk. – 2011. – С. 17.