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DIRIXLE PROBLEM FOR A (z) -HARMONIC FUNCTION

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Annotation
As is well known, the classical Poisson formula is the simplest and most important example of solving the Dirichlet problem in the class of Harmonic Functions.
The following Drixle problem for the harmonic functions $A(z)$ is considered. Dirichli issue. The bounded $G \in D$ field is continuous at the given ∂G boundary $\varphi(\xi)$. $A(z)$ G is a harmonic function in the field $\overline{G}$ and continuous. Function $u(z) \in h_A(G) \cap C(\overline{G})$: $u|_{\partial G} = \varphi$
As is well known, the classical Poisson formula is the simplest and most important example of solving the Dirichlet problem in the class of Harmonic Functions.

Theorem (1) (analogue of Poisson's formula for the $A(z)$ -harmonic function)
If the function $\Psi(t)$ is a continuous function at the boundary $D \in L(a, r)$ limnescata, then the Dirichlet problem is defined as

$$u(z) = \frac{1}{2\pi R} \oint_{|\omega(\xi,z)|=R} \varphi(\xi) \frac{R^2 - |\Psi(a,z)|^2}{|\Psi(t,z)|^2} |d\xi + A(\xi) d\bar{\xi}| \quad (1)$$

Proof. Here $f(\xi, z) = \frac{\Psi(a,\xi)+\Psi(a,\xi)}{\Psi(z,t)}$ function A (z) -analytic function

$z \in L(a, r)$, $T \in \partial L(a, R)$, $\xi \in \partial L(a, r)$ $\prod(\xi, z) = \frac{1}{2\pi} Ref(\xi, z) \in h_A L(a, R)$

$$\prod(\xi, z) = \frac{1}{2\pi} Ref(\xi, z) + \bar{f}(z, \xi) = \frac{1}{2\pi}\left[\frac{\Psi(a,\xi) + \Psi(a,z)}{\Psi(a,t) - \Psi(a,z)} + \frac{\bar{\Psi}(a,\xi) + \bar{\Psi}(a,z)}{\bar{\Psi}(a,\xi) - \bar{\Psi}(a,z)}\right]$$

$$= \frac{1}{2\pi}\left[\frac{|\Psi(a,\xi)|^2 - |\omega(a,z)|^2}{|\Psi(\xi,z)|^2}\right] = \frac{1}{2\pi}\left[\frac{R^2 - |\Psi(a,z)|^2}{|\Psi(\xi,z)|^2}\right]$$

As a result, we show that the function $u \in h_A(L(a, R))$ is $\bar{L}(a, R)$, and $u|_{\partial L(a,R)} = \varphi$. We use the following two facts

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1. $\frac{1}{2\pi R}\oint_{|\omega(\xi,z)|=R}\varphi(t)\frac{R^2-|(a,z)|^2}{|(\xi,z)|^2}\left|d\xi+A(\xi)d\bar{\xi}\right|=1$

Then $z\to\xi_0\in\partial L(a,R)$, and the function $\xi\neq\xi_0\prod(\xi,z)\to 0$ equally for an arbitrary are $\gamma_\delta=\partial L(a,R)\backslash U(\xi,\delta)$. $\Psi(a,\xi)=Re^{it}$ can be written as follows

$$\left|dz+A\overline{dz}\right|=\left|\frac{\partial\Psi}{\partial z}dz+\frac{\partial\Psi}{\overline{\partial z}}\overline{dz}\right|=|d\Psi(\xi,a)|=\left|dRe^{it}\right|=\left|Rie^{it}dt\right|=Rdt,$$

$$\frac{1}{2\pi R}\oint_{|\Psi(\xi,a)|=R}\varphi(\xi)\frac{R^2-|\Psi(a,z)|^2}{|\Psi(t,z)|^2}\left|d\xi+A(\xi)d\bar{\xi}\right|=\frac{1}{2\pi}\int_0^{2\pi}\varphi(\xi)\frac{R^2-|\Psi(a,z)|^2}{|\Psi(\xi,z)|^2}d\xi$$

$$\Delta=u(z)-\frac{1}{2\pi}\int_0^{2\pi}\varphi(\xi)\frac{R^2-|\Psi(a,z)|^2}{|\Psi(\xi,z)|^2}d\xi=\frac{1}{2\pi}\int_0^{2\pi}(u(z)-\varphi(\xi))\frac{R^2-|\Psi(a,z)|^2}{|\Psi(\xi,z)|^2}d\xi$$

from the continuity of $\varphi(\xi)$ and $\xi_0\in\partial L(a,R)$: $\Psi(a,\xi_0)=Re^{it_0}$ Given that

$\forall\varepsilon>0\ \exists\delta>0\ \ \forall\xi=\partial L(a,R)\backslash U(a,\delta)\Rightarrow|\varphi(\xi)-(\varphi(\xi_0)<\varepsilon$.

We calculate as follows. $I_1=\{t\in[0,2\pi]: Re^{it}\in\ \partial L(a,R)\backslash U(\xi_0,\delta)\}$

$$I_2=\{t\in[0,2\pi]: Re^{it}\in\ \partial L(a,R)\cap U(\xi_0,\delta)\}$$

In this we evaluate

$$I_1\cup I_2=[0,2\pi]$$

$$\Delta=\frac{1}{2\pi}\int_{I_1}(u(z)-\varphi(\xi))\frac{R^2-|\Psi(a,z)|^2}{|\Psi(\xi,z)|^2}d\xi+\frac{1}{2\pi}\int_{I_2}(u(z)-\varphi(\xi))\frac{R^2-|\Psi(a,z)|^2}{|\Psi(o,z)|^2}d\xi=J_1+J_2\quad J_2$$

$$|J_2|\leq\frac{1}{2\pi}\int_{I_1}(u(z)-\varphi(\xi))\frac{R^2-|\Psi(a,z)|^2}{|\Psi(\xi,z)|^2}d\xi\leq\frac{1}{2\pi}\int_0^{2\pi}\varepsilon\frac{R^2-|\Psi(a,z)|^2}{|\Psi(\xi,z)|^2}d\xi<\varepsilon$$

Suppose we express $\Psi(a,z)=re^{i\theta}$ ith in the form $|\theta-t_0|<\frac{\delta}{2}$. For all $t\in I_1$ we find $\rho\in(R-r,R)$ according to the above 2 facts and there is an inequality $\prod(\xi,z)<\varepsilon$

Then for all z $|\Psi(a,z)|=r>R-\rho$, $|\theta-t_0|<\frac{\delta}{2}$

$$|J_1|\leq\frac{1}{2\pi}\int_{I_1}(u(z)-\varphi(\xi))\frac{R^2-|\Psi(a,z)|^2}{|\Psi(\xi,z)|^2}d\xi\leq\frac{\varepsilon}{2\pi}2\max|\varphi(\xi)|\int_0^{2\pi}\frac{R^2-|\Psi(a,z)|^2}{|\Psi(\xi,z)|^2}d\xi<2\varepsilon\max|\varphi(\xi)|$$

We generate the following expression. $|\Delta|<\varepsilon(1+2\max\{\varphi(\xi)\}$ and $\lim\limits_{z\to\xi_0}u(z)=\varphi(\xi_0)$. The theorem is proved.

LITERATURE

1.Шабат Б.В.,Введение в комплексный анализ,часть 1, М., “Наука”, 1985г.

2.Л.А.Айзенберг. “Формулы Карлемана в комплексном анализе” Новосибирск «Наука» 1990г.247с

3.Arbuzov E.V, Bukhgeim A.L. ‘Carleman’s formulas for A-analytic functions in a half-plane’// J.Inv.III-Posed Problems, 1997, v. 5, N.6, pp.491-505.