



## PRODUCTING A GEOMETRIC MODEL OF CONSTRUCTING THE FRACTALS IN THE SHAPE OF TREE AND PENTAGONAL

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### Abstract

This article is consisted of the study of problems in fractal construction, the concept of fractal measurement and their calculation methods, the main directions and prospects of their usage and the analysis in the current state of fractal construction methods. By the basic principles of fractal theory, analytical and R-function methods, geometric models for complex fractal images in the shape of tree and with pentagonal were produced and brought up results.

**Keywords:** R-function, fractal, analytical method, fractal in the shape of tree.

### Introduction

It is obvious that geometric fractals are formalized, starting from an initiator shape using a basic image. Determinized fractals are expressed in a recursive process. In deterministic fractals, self-similarity appears in all orders. Such fractals are iterated 4-6 times to obtain clear images.

### Problem Setting and Solution Methods

Based on V. L. Rvachev's R-function method, we construct the fractal equation in the shape of tree.

Firstly, we take into consideration the construction of in the shape of tree equation from circles. If the ends of the interval are the points  $(x_1, y_1)$  and  $(x_2, y_2)$ , by given points  $(x_1, y_1)$  and  $(x_2, y_2)$  we construct the equation of a free straight line [ 3-5 ]





$$f(x_1, y_1, x_2, y_2, x, y) = \left( \left( \frac{1}{2}((x_2 - x_1) \cos \left( \arctan \left( \frac{y_2 - y_1}{x_2 - x_1} \right) \right) + (y_2 - y_1) \sin \left( \arctan \left( \frac{y_2 - y_1}{x_2 - x_1} \right) \right) \right) \right)^2 - \left( (x - x_1) \cos \left( \arctan \left( \frac{y_2 - y_1}{x_2 - x_1} \right) \right) + (y - y_1) \sin \left( \arctan \left( \frac{y_2 - y_1}{x_2 - x_1} \right) \right) - \left( \frac{1}{2}(x_2 - x_1) \cos \left( \arctan \left( \frac{y_2 - y_1}{x_2 - x_1} \right) \right) + (y_2 - y_1) \sin \left( \arctan \left( \frac{y_2 - y_1}{x_2 - x_1} \right) \right) \right)^2 \geq 0 \right) \wedge_0 \wedge_0 \left( a^2 - \left( - (x - x_1) \sin \left( \arctan \left( \frac{y_2 - y_1}{x_2 - x_1} \right) \right) + (y - y_1) \cos \left( \arctan \left( \frac{y_2 - y_1}{x_2 - x_1} \right) \right) \right)^2 \geq 0 \right) \quad (1)$$

b where  $a$  is the height of the gap (the height of the gap is equal to  $2a$ ).

If  $k$  is even number, then  $\varphi_0 = 0$ , otherwise  $\varphi_0 = \frac{\alpha}{2}$ .

In  $n=1$  we get the following equation:  $\alpha = \frac{2\pi}{k}$

$$\omega_1(x, y) = f(0, 0, R \cos(\varphi_0 + 0), R \sin(\varphi_0 + 0), x, y) \vee_0$$

$$\vee_0 f(0, 0, R \cos(\varphi_0 + \alpha), R \sin(\varphi_0 + \alpha), x, y) \vee_0$$

$$\vee_0 f(0, 0, R \cos(\varphi_0 + 2\alpha), R \sin(\varphi_0 + 2\alpha), x, y) \vee_0 \dots \vee_0 \quad (2)$$

$$\vee_0 f(0, 0, R \cos(\varphi_0 + (k-1)\alpha), R \sin(\varphi_0 + (k-1)\alpha), x, y)$$

at  $n=2, 3, 4 \dots$

$$\alpha = \frac{2\pi}{k^{n-1}}; k_1 = -[k/2]; R_{n-1} = 2R(1 - \frac{1}{2^{n-1}}); R_n = 2R(1 - \frac{1}{2^n});$$

$R_n - n$  - in iteration, the radius of the circular boundaries ( $R_1 = R$ ).

If  $k$  is even, then  $k_2 = [k/2]$ , otherwise  $k_2 = [k/2] - 1$ .

$[x]$  is the integer part of the number  $x$ .

Using the iteration procedure, we get:

$$\omega_{n+1}(x, y) = f(R_{n-1} \cos(\varphi_0 + \alpha), R_{n-1} \sin(\varphi_0 + \alpha),$$

$$R_n \cos \left( \varphi_0 + \alpha + \frac{(\varphi_0 + k_1 \alpha)}{k} \right), R_n \sin \left( \varphi_0 + \alpha + \frac{(\varphi_0 + k_1 \alpha)}{k} \right), x, y) \vee_0$$

$$\vee_0 f(R_{n-1} \cos(\varphi_0 + \alpha), R_{n-1} \sin(\varphi_0 + \alpha),$$



$$\begin{aligned}
 &R_n \cos(\varphi_0 + \alpha + \frac{(\varphi_0 + (k_1 + 1)\alpha)}{k}), R_n \sin(\varphi_0 + \alpha + \frac{(\varphi_0 + (k_1 + 1)\alpha)}{k}), x, y) \vee_0 \\
 &\vee_0 f(R_{n-1} \cos(\varphi_0 + \alpha), R_{n-1} \sin(\varphi_0 + \alpha), \\
 &R_n \cos(\varphi_0 + \alpha + \frac{(\varphi_0 + (k_1 + 2)\alpha)}{k}), R_n \sin(\varphi_0 + \alpha + \frac{(\varphi_0 + (k_1 + 2)\alpha)}{k}), x, y) \vee_0 \dots \vee_0 \\
 &\vee_0 f(R_{n-1} \cos(\varphi_0 + \alpha), R_{n-1} \sin(\varphi_0 + \alpha), R_n \cos(\varphi_0 + \alpha + \frac{(\varphi_0 + k_2\alpha)}{k}), R_n \sin(\varphi_0 + \alpha + \frac{(\varphi_0 + k_2\alpha)}{k}), x, y) \\
 &\omega_{nx2}(x, y) = f(R_{n-1} \cos(\varphi_0 + 2\alpha), R_{n-1} \sin(\varphi_0 + 2\alpha), R_n \cos(\varphi_0 + 2\alpha + \frac{(\varphi_0 + k_1\alpha)}{k}), \\
 &R_n \sin(\varphi_0 + 2\alpha + \frac{(\varphi_0 + k_1\alpha)}{k}), x, y) \vee_0 \\
 &\vee_0 f(R_{n-1} \cos(\varphi_0 + 2\alpha), R_{n-1} \sin(\varphi_0 + 2\alpha), R_n \cos(\varphi_0 + 2\alpha + \frac{(\varphi_0 + (k_1 + 1)\alpha)}{k}), \\
 &R_n \sin(\varphi_0 + 2\alpha + \frac{(\varphi_0 + (k_1 + 1)\alpha)}{k}), x, y) \vee_0 \\
 &\vee_0 f(R_{n-1} \cos(\varphi_0 + 2\alpha), R_{n-1} \sin(\varphi_0 + 2\alpha), R_n \cos(\varphi_0 + 2\alpha + \frac{(\varphi_0 + (k_1 + 2)\alpha)}{k}), \\
 &R_n \sin(\varphi_0 + 2\alpha + \frac{(\varphi_0 + (k_1 + 2)\alpha)}{k}), x, y) \vee_0 \dots \vee_0 \\
 &\vee_0 f(R_{n-1} \cos(\varphi_0 + 2\alpha), R_{n-1} \sin(\varphi_0 + 2\alpha), R_n \cos(\varphi_0 + 2\alpha + \frac{(\varphi_0 + k_2\alpha)}{k}), \\
 &R_n \sin(\varphi_0 + 2\alpha + \frac{(\varphi_0 + k_2\alpha)}{k}), x, y) \quad (3)
 \end{aligned}$$

For  $1 \leq i \leq k^{n-1}$  we get :

$$\begin{aligned}
 &\omega_{nxi}(x, y) = f(R_{n-1} \cos(\varphi_0 + i\alpha), R_{n-1} \sin(\varphi_0 + i\alpha), R_n \cos(\varphi_0 + i\alpha + \frac{(\varphi_0 + k_1\alpha)}{k}), \\
 &R_n \sin(\varphi_0 + i\alpha + \frac{(\varphi_0 + k_1\alpha)}{k}), x, y) \vee_0 \\
 &\vee_0 f(R_{n-1} \cos(\varphi_0 + i\alpha), R_{n-1} \sin(\varphi_0 + i\alpha), \\
 &R_n \cos(\varphi_0 + i\alpha + \frac{(\varphi_0 + (k_1 + 1)\alpha)}{k}), R_n \sin(\varphi_0 + i\alpha + \frac{(\varphi_0 + (k_1 + 1)\alpha)}{k}), x, y) \vee_0 \\
 &\vee_0 f(R_{n-1} \cos(\varphi_0 + i\alpha), R_{n-1} \sin(\varphi_0 + i\alpha), \\
 &R_n \cos(\varphi_0 + i\alpha + \frac{(\varphi_0 + (k_1 + 2)\alpha)}{k}), R_n \sin(\varphi_0 + i\alpha + \frac{(\varphi_0 + (k_1 + 2)\alpha)}{k}), x, y) \vee_0 \dots \vee_0 \\
 &\vee_0 f(R_{n-1} \cos(\varphi_0 + i\alpha), R_{n-1} \sin(\varphi_0 + i\alpha), \\
 &R_n \cos(\varphi_0 + i\alpha + \frac{(\varphi_0 + k_2\alpha)}{k}), R_n \sin(\varphi_0 + i\alpha + \frac{(\varphi_0 + k_2\alpha)}{k}), x, y)
 \end{aligned}$$



$$\omega_n(x, y) = \omega_{n-1}(x, y) \vee_0 \omega_{nx1}(x, y) \vee_0 \omega_{nx2}(x, y) \vee_0 \dots (4)$$
$$\vee_0 \omega_{nxi}(x, y) \vee_0 \dots \vee_0 \omega_{nxk^{n-1}}(x, y).$$

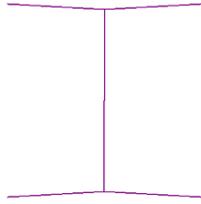
In the previous formulas  $k=2, 3, 4, 5, \dots$

$R_n$  for all lines it is possible to draw an outer circle with a radius ( $n$ -order iteration).

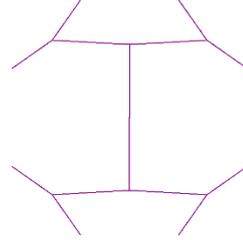
Calculation results for different measurements of  $n$  and  $k$  are presented in Figure 1.



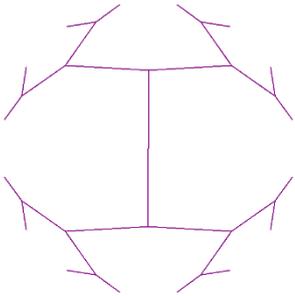
$n=1, k=2$



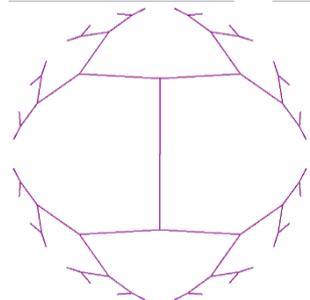
$n=2, k=2$



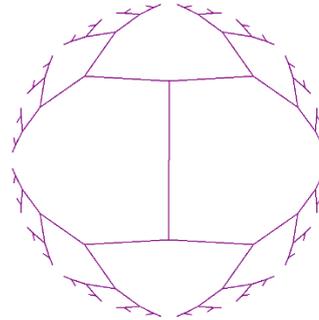
$n=3, k=2$



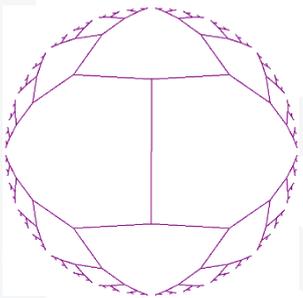
$n=4, k=2$



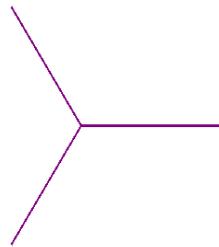
$n=5, k=2$



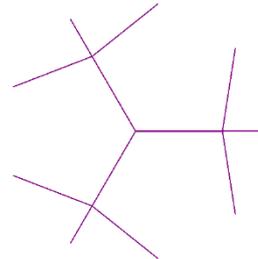
$n=6, k=2$



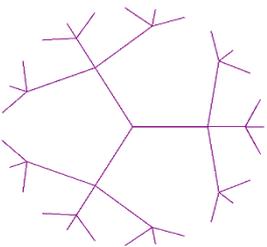
$n=7, k=2$



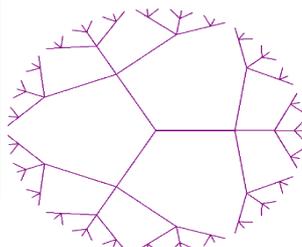
$n=1, k=3$



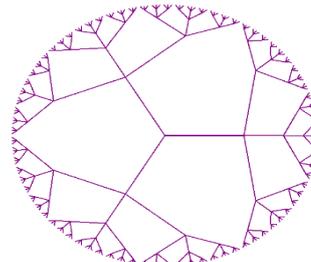
$n=2, k=3$



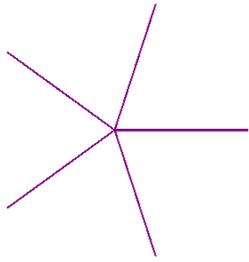
$n=3, k=3$



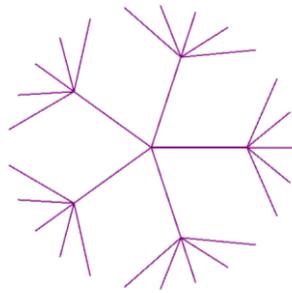
$n=4, k=3$



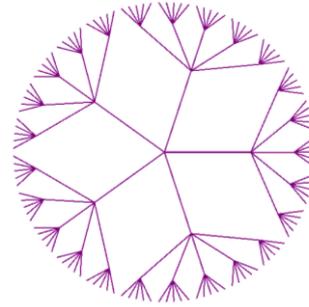
$n=5, k=3$



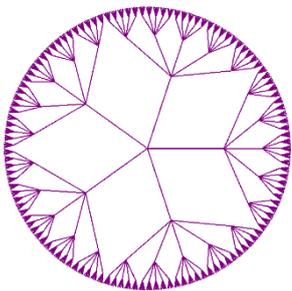
$n=1, k=5$



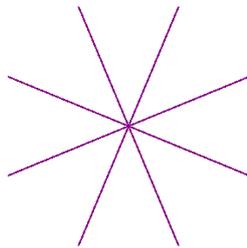
$n=2, k=5$



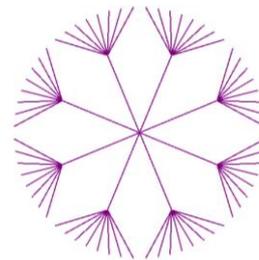
$n=3, k=5$



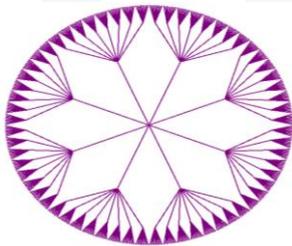
$n=4, k=5$



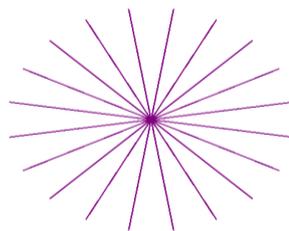
$n=1, k=8$



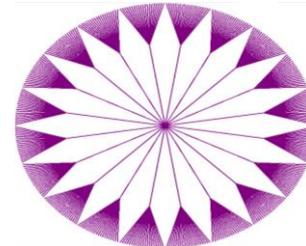
$n=2, k=8$



$n=3, k=8$



$n=1, k=20$

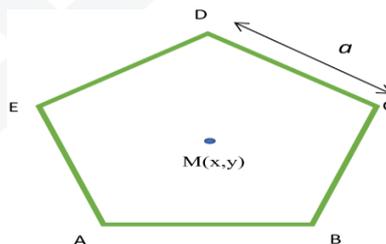


$n=2, k=20$

### 1 . Fractals in the shape of tree

Algorithm for constructing fractals consisting of pentagons  $i \in [1,6,7]$  :

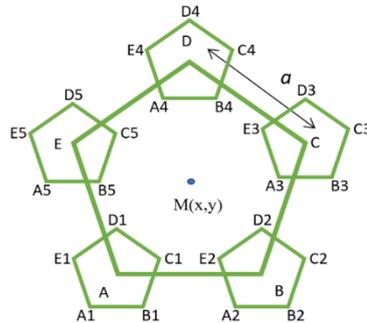
a) When we construct fractals of this type, a pentagon, being equal to "  $a$  " is drawn and its center is determined (its center is the intersection of the heights passed from its ends). The coordinate of this point is determined.



### 2 . In steps 1 and 2, pentagonal fractals



In the next step, the edges of the resulting pentagons are reduced by  $2/5$  times and placed at the ends of the first pentagon.



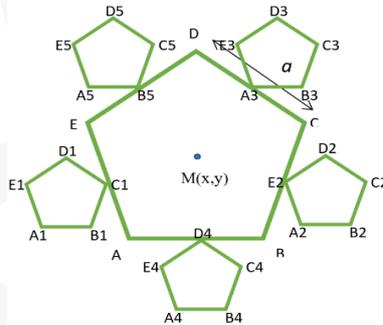
$$\begin{aligned}
 A(x, y) &= (x - a / 2, y - a * 2 / 3) \\
 B(x, y) &= (x - a * 3 / 5, y - a * 2 / 3) \\
 C(x, y) &= (x + a, y + a / 2) \\
 D(x, y) &= (x - a * 8 / 9, y + a / 2) \\
 E(x, y) &= (x + a / 17, y + a * 10 / 9)
 \end{aligned}$$

### 3 . A pentagonal fractal in iteration 3

This process is repeated  $n$  times ( see Figure 5 ( a ) ), and the formula for calculating the number of pentagons can be written as the following:

$$1 + 5 + 25 + 125 + \dots + 5^{n-1} = \sum_{i=1,2,\dots}^n 5^{i-1} \quad (5)$$

b) To construct pentagons of this type, as in a ) in the first step, a pentagon whose side is equal to "  $a$  " is drawn. In the second step, its center is determined (its center is the point where the heights passed from the ends intersect). The coordinate of this point is determined ( $M(x,u)$ ). The middle of the sides  $A B$  ,  $B C$  ,  $C D$  ,  $D E$  and  $EA$  of the pentagon is determined, and pentagons are placed that are  $2/5$  times smaller than the size of the previous step, meeting the determined points.

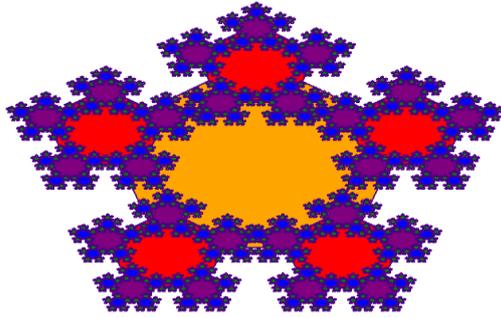


$$\begin{aligned}
 AE(x, y) &= (x - a * 11 / 10, y - a / 8) \\
 BC(x, y) &= (x + a * 6 / 5, y - a / 8) \\
 CD(x, y) &= (x + a * 4 / 5, y + a * 9 / 8) \\
 DE(x, y) &= (x - a * 2 / 3, y + a * 9 / 8) \\
 AB(x, y) &= (x, y - a)
 \end{aligned}$$

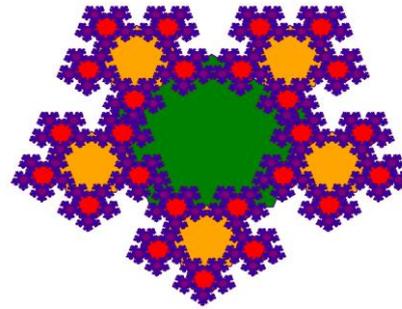
### 4 . A pentagonal fractal in iteration 3

This process is repeated  $n$  times ( see Figure 5 ( b ) ), the formula for calculating the number of pentagons can be written as the following:

$$1 + 5 + 25 + 125 + \dots + 5^{n-1} = \sum_{i=1,2,\dots}^n 5^{i-1}$$



*a*



*b*

5 . Pentagonal fractals in iteration 6

### Conclusion:

Various methods of developing a simple algorithmic function that generates a fractal model considered. Using the principles of fractal geometry, fractals based on some algorithms to create them through various geometrical permutations of some structured sets, mentioned above were generated.

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