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PARAMETRIC LINEAR EQUATIONS AND METHODS FOR THEIR SOLUTION

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Abstract

In this article, problems related to parametric linear equations and the algorithm for solving them are given, and by studying and solving them, students of secondary schools can learn the skills of independent solving of parametric equations and introduce various methods used in the process of solving parametric problems. intended.

Keywords: variable, parameter, equation, solution, algorithm, function, graph, coordinate system, values.

ПАРАМЕТРИЧЕСКИЕ ЛИНЕЙНЫЕ УРАВНЕНИЯ И МЕТОДАХ ИХ РЕШЕНИЯ

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Аннотация:

В данной статье представлены задачи, связанные с параметрическими линейными уравнениями и алгоритм их решения, а зная и изучая их, учащиеся общеобразовательных школ могут овладеть навыками самостоятельного решения параметрических уравнений и познакомить с различными методами, используемыми в процессе решения параметрических задач.

Ключевые слова: переменная, параметр, уравнение, решение, алгоритм, функция, график, система координат, значения.

Solving equations involving parameters is perhaps one of the most difficult parts of elementary mathematics. The reason for this is that at school they often try to form skills and abilities to solve certain standard problems related to the technique of algebraic transformations. Parametric problems are a different type of problem. Their solution usually requires flexibility of thinking, logic in thinking, ability to analyze the situation well and completely. Experience shows that students who have mastered the methods of solving parameter problems successfully solve other problems. Therefore, problems related to parameters are of diagnostic and prognostic importance.

For several years, many universities have included parametric equations in examples and problems of entrance exams (olympiads). But so far, the parametric equation remains the most "inconvenient" for applicants. Because all schoolchildren start solving these tasks "from afar" and only in some cases do the solution correctly.

A parameter (from the Greek "parametrion" - measurement) is a quantity whose values serve to separate the elements of a certain set from each other. Parameters are used to study many systems and processes in real life. In particular, in physics, temperature, time, etc. can act as parameters. In mathematics, parameters are introduced to define a set of certain objects. For example, the equation $x^2 + y^2 = c^2$ is a set of concentric circles located at the coordinate origin, where c is a parameter.

Analyzing the possible ways of solving specific examples of solving parametric linear equations, and creating a known algorithm for solving such problems, the "complex" parameter becomes "simple" and it greatly helps in the initial stages of learning to solve parametric equations.

An equation of the form $f(a)\cdot x = g(a)$, where $f(a)$, $g(a)$ are analytical expressions, is called linear with respect to the variable $-x$ with parameter $-a$. Solving such an equation means finding the values of the variable $-x$ that satisfy this equation for each admissible value of the parameter $-a$. Solving such linear parametric equations usually involves the following three steps.

1. If $f(x) \neq 0$, then the equation has a unique solution of the form $x = \frac{g(a)}{f(a)}$.

2. If $f(x) = 0$ and $g(x) \neq 0$, then a false equality $0 \cdot x = g(a)$ appears at an arbitrary value of $-x$, that is, the equation has no roots.

3. If $f(x) = 0$ and $g(x) = 0$, then the equation takes the form $x \cdot 0 = 0$ and it is valid for any real value of $-x$.

Taking into account these stages of solving parametric linear equations, it is convenient to divide them into groups in the process of solving equations, in turn, depending on the conditions of the problem. Because, in this case, the students remember the ways to solve the problem faster, depending on the condition of the problem. Parametric linear equations can usually be divided into the following main groups depending on the condition of the problem:

1. Equations required to be solved for an arbitrary value of the parameter or for -*a* set of predefined values of the parameter.

Example 1. Solve the equation $x+2=ax$ with respect to parameter -*a*.
Solution: We solve the equation with respect to -*x*. $x-ax=-2$ or $(1-a)x=-2$
a). If *a=1*, then the equation looks like $0\cdot x=-2$ and has no solution.
b).If $a\neq 1$, then the equation has a unique solution $x=\frac{2}{a-1}$.
So, if *a=1*, then it has no solution; if $a\neq 1$, then the equation has a unique solution $x=\frac{2}{a-1}$.

Example 2. Solve the equation with respect to parameter -*a* and -*b*.
$$\frac{ax-1}{x-1}+\frac{b}{x+1}-\frac{a(x^2+1)}{x^2-1}=0$$

Solution: a). It is known that the equation makes sense only when $x\neq\pm 1$.
b). let $x\neq\pm 1$. If we perform the same form changes in the equation, we will create $(ax-1)(x+1)+b(x-1)=a(x^2+1)$ or $x(a+b-1)=a+b+1$ with equal strength.
Here, if $a+b=1$, the equation has no solution. If $a+b\neq 1$, then $x=\frac{a+b+1}{a+b-1}$ is the solution.
v). The condition $x\neq -1$ results in $a+b+1=-(a+b-1)$ or $a+b\neq 0$.
g). In the equation $x=1$, $a+b+1=a+b-1$ is not fulfilled at any value of parameters -*a* and -*b*.
So it can be concluded that if $a+b\neq 0$ and $a+b\neq 1$, then $x=\frac{a+b+1}{a+b-1}$
if $a+b=0$ or $a+b=1$, then $x\in\varnothing$, that is, there is no solution.

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2. Equations where it is required to determine the number of solutions of the equation depending on the parameter values.

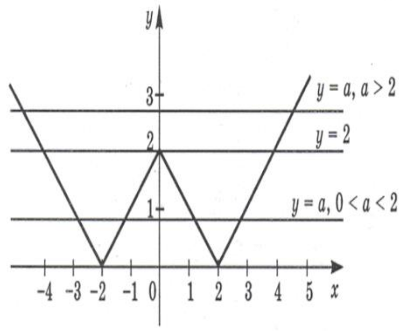
Example 3. How many roots does the equation $||x|-2|=a$ have depending on the parameter -*a*?

Solution: We make the graphs of the functions $y=||x|-2|$ and $y=a$ in the *OXY* coordinate system. It can be seen from the drawing:

a) If *a*=0, then the straight line $y=a$ overlaps with the *OX* axis and $y=||x|-2|$ has two points in common with the graph of the function, so the given equation has two roots.

b) b) If $0<a<2$, then the straight line $y=a$ has four points in common with the graph of the function $y=||x|-2|$, so the given equation has four roots.

c) If $a=2$, then the straight line $y=a$ has three points in common with the graph of the function $y=||x|-2|$, so the given equation has three roots.



d) If $a>2$, then the straight line $y=a$ has two points in common with the graph of the function $y=||x|-2|$, so the given equation has two roots.

So if $a<0$ the equation has no roots, if $a=0$ or $a>2$ the equation has two roots, if $a=2$ it has three roots and finally if $0<a<2$ it has four roots as long as it happens.

Example 4. Find all values of the parameter $-a$ for which the equation $a|x-1|=x+2$ has a unique solution.

Solution: To switch from the given equation to the module-free equation, the following two cases $x\geq 1$ and $x<1$ must be considered.

1). let $x\geq 1$. The given equation is in the form $a(x-1)=x+2$, and after changing the form $x(a-1)=a+2$.

It can be seen that the equation has no solution at $a=1$.

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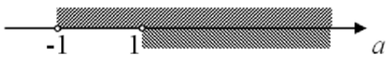


If $a \neq 1$, then $x = \frac{a+2}{a-1}$ is the solution of the given equation under the condition $x \geq 1$. We solve the inequality $\frac{a+2}{a-1} \geq 1$. After changing the form, we find the solution $\frac{3}{a-1} \geq 0$ and $a \in (1;+\infty)$. So, the equation has a unique root $a \in (1;+\infty)$ when $x = \frac{a+2}{a-1}$.

2). let $x < 1$. The given equation is in the form $a(1-x) = x+2$, and after changing the form $x(a+1) = a-2$.

It can be seen that the equation has no solution at $a = -1$.

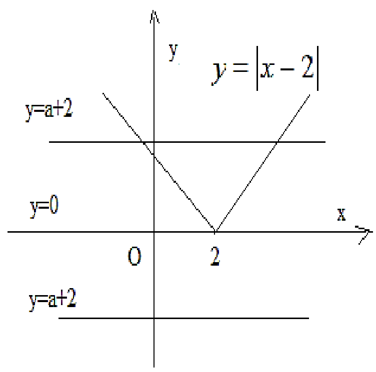
If $a \neq -1$, then $x = \frac{a-2}{a+1}$, this is the solution of the given equation under condition $x < 1$, we solve the inequality $\frac{a-2}{a+1} < 1$. After changing the form, we find the solution $\frac{-3}{a+1} < 0$ and $a \in (-1;+\infty)$. So, the equation has a unique root $x = \frac{a-2}{a+1}$ when $a \in (-1;+\infty)$.

We determine the values of the parameter a, which will have a solution to the given equation on the number axis.

As you can see from the diagram (marked with a line), the equation $x = \frac{a-2}{a+1}$ has a single root when $a \in (-1;1]$. (when $a \in (1;+\infty)$, the equation has two roots $x = \frac{a-2}{a+1}$ and $x = \frac{a+2}{a-1}$).

Example 5. Find the values of the parameter -*a* for which the equation $|x-2| = a+2$ has a unique solution?

Solution: We make graphs of functions $y = |x-2|$ and $y = a+2$ in the *OXY* coordinate system. It can be seen from the figure that,



a) If $a+2 < 0$ i.e. $a < -2$, the graphs of the functions do not intersect and the equation has no solution.

b) If $a+2 = 0$, i.e. $a = -2$, the graphs of the functions intersect at only one point and the equation has a unique solution.

c) If $a+2 > 0$ i.e. $a > -2$, the graphs of the functions intersect at two points and the equation has two solutions.

So, the given equation has -*a* unique solution only at the value of parameter $a = -2$.

3. It is necessary to find the values of the parameter so that the specified equations have a given number of solutions (in particular, they do not have a solution, they have an infinite number of solutions).

Example 6. Find such values of parameter -*m* that $m^2x-m^2+6=4x+m$ equation:
a). have a unique solution. b). do not have a solution. c). have infinitely many solutions.

Solution: We write the given equation in the form $(m^2-4)x=m^2+m-6$.

a).The equation has a unique solution $x=\dfrac{m^2+m-6}{m^2-4}$ when $m^2-4\neq 0$, i.e. $m\neq\pm 2$.

b). If the condition $\begin{cases} m^2-4=0 \\ m^2+m-6\neq 0 \end{cases} \Rightarrow \begin{cases} m=\pm 2 \\ m\neq 2, m\neq -3 \end{cases}$ is fulfilled, the equation has no solution, from which $m=-2$ follows.

v). In order for the equation to have infinitely many solutions, $\begin{cases} m^2-4=0 \\ m^2+m-6=0 \end{cases} \Rightarrow \begin{cases} m=\pm 2 \\ m=2, m=-3 \end{cases}$ conditions must be fulfilled, from which $m=2$ follows.

So, the equation has a unique solution at $m\in(-\infty;-2)\cup(-2;2)\cup(2;+\infty)$, no solution at $m=-2$, and infinitely many solutions at $m=2$.

Example 7. At what values of parameter -*a* the equations $x^2+ax+1=0$ and $x^2+x+a=0$ have one common root.

Solution: If x_1- is a general solution of the given equations, then the equations $x_1{}^2+ax_1+1=0$ and $x_1{}^2+x_1+a=0$ are valid. If we find $a=-x_1^2-x_1$ from the second equation and put it in the first equation, we get $x_1^2-x_1(x_1^2+x_1)+1=0$ or $x_1^3=1$, from which $x_1=1$ and $a=-2$ are derived.

So, only when $a=-2$, the given equations have a unique common solution $x_1=1$.

Result:

So, the algorithm for solving parametric linear equations can be understood as follows:

1. to standardize the equation (if necessary).
2. finding a set of values that the parameter can accept; the equation does not have a solution for all values of the parameter that do not belong to this set.

3. finding all values of the parameter where the coefficient in front of the unknown becomes zero; among these values, to find intervals where the free term of the parameter becomes zero and is different from zero.
4. solving the linear equations formed in the intervals of the parameter found in step 3
5. find the general solution of the equation for all remaining values of the parameter by dividing both sides by the coefficient in front of the unknown.
6. writing the answer, taking into account the solutions found in the intervals for all values of the parameter.

Summary

The comments and solved problems in this article help schoolchildren to independently learn some methods of solving parametric equations and inequalities, and teachers to work on these topics in class or extracurricular activities (elective courses). We hope that it will help in the systematic organization of.

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