



FINDING THE OPTIMAL SOLUTION TO ALGORITHMIC PROGRAMMING ISSUES THROUGH GRAPHS

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Annotation

This article discusses the problem of finding the optimal solution to economic processes using mathematical modeling, specifically the linear programming problem, through graphical methods.

Keywords: Linear, graph, mathematics, model, process, function, Cartesian, object. Mathematics has long been known for its applications in disciplines such as astronomy, mechanics, and physics. In the 1940s, the discovery of electronic computing machines, particularly the advancement of information technologies, greatly expanded the possibilities of mathematical methods. On one hand, it increased the potential of mathematical techniques, while on the other hand, it led to a significant expansion of their applications. Presently, it is difficult to find a field where mathematics is not being utilized. It plays a universal role in the theoretical and applied approaches of many disciplines. Mathematics is understood as a subject that encompasses all our knowledge about nature, providing mathematical models for real processes in nature and society.

Mathematical models of economic objects are expressed through equations, inequalities, logical connections, graphs, and other means of representation. This representation should incorporate the interrelationships between the constituent elements of the object being studied. The term "model" indicates that it provides a conditional description of the economic object. By organizing the model, one can obtain new information about the object and find the best (optimal) solutions that correspond to various conditions.

Economists use simplified and formalized economic models to understand various economic processes. Examples of economic models include supply and demand models, firm models, Leontief models, economic growth models, models of market equilibrium in commodity markets, and others. When constructing a model, important factors that determine the processes in the object being modeled are considered, while insignificant ones are not included in the model structure.

In today's complex production process and the expanding nature of market relations, it is essential to analyze each task and draw accurate conclusions based on them. Mathematical programming plays a fundamental role in formalizing such theories.





Mathematical programming is a tool that primarily deals with solving economic problems that have multiple possible solutions to find the best, optimal solution according to the objective. Mathematical programming encompasses linear programming, nonlinear programming, and dynamic programming. It is important to note that there is no universal method for solving linear programming problems. Existing methods have mainly been developed and tailored for specific instances of linear programming problems.

Let's consider the general problem of mathematical optimization in terms of programming.

$$F = c_1x_1 + c_2x_2 + \dots + c_nx_n \quad (1)$$

continuous function and

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases} \quad (2)$$

$$x_j \geq 0, j = 1, 2, \dots, n \quad (3)$$

Given a system of constraints involving continuous variables, and objective (1) is to find values for these variables that satisfy the constraint system (2) and optimize the objective function (1) to reach a minimum (or maximum) value.

We will solve the problem of mathematical optimization graphically to find the optimal solution.

The geometric representation (visualization) of the linear programming problem is possible when dealing with discrete cases. Let's consider the linear programming problem given as follows:

Example;

$$\begin{cases} 5x_1 + 3x_2 \geq 15 \\ 3x_1 - 5x_2 \leq 15 \\ x_1 + 2x_2 \leq 10 \\ x_1 \geq 0, x_2 \geq 0 \end{cases}$$

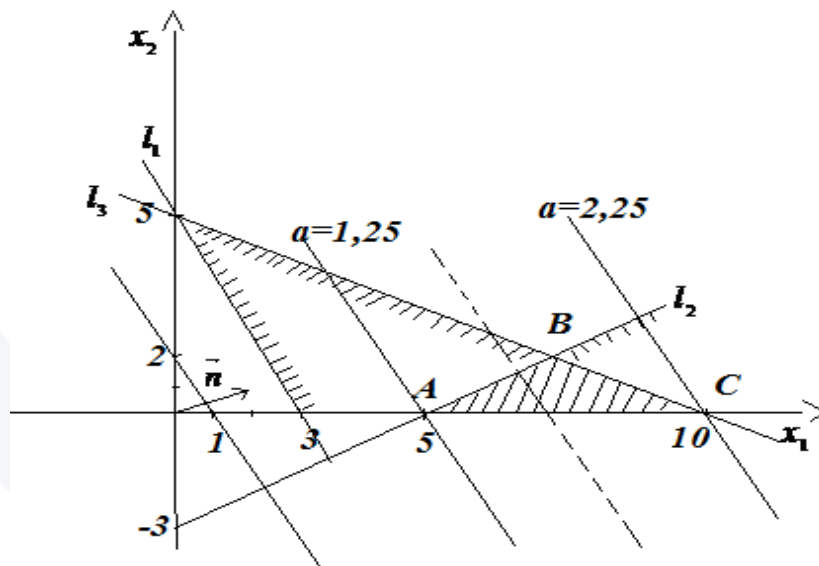
collect the maximum and minimum value of the characteristics of satisfaction of the conditions.

Solution: Let's determine the region that satisfies the inequalities in the Cartesian coordinate system. The set of solutions that satisfies the given system of inequalities forms a region ABC, which represents the feasible solutions.

The objective is to find the points within this region where the objective function attains its maximum and minimum values. We will first find the location of the points that yield the same values for the objective function.



To do this, we define a constant "a" and form an equation, which represents a line that accepts the same values for "a." Instead of assigning various values to "a," we create parallel lines. Each of these lines is called an "equi-level line" (or contour line), as the function takes the same values along these lines. We examine the normal vector of these lines. We draw a perpendicular line (one of the contour lines) to this vector and extend it until it intersects with the set of feasible solutions. In this case, if we need to find the maximum value in the problem, we shade the region in the opposite direction to the direction of the vector, and if we need to find the minimum value, we shade the region in the direction of the vector.



1-Figure

Based on the given information in the first figure, it can be observed that the function F attains its minimum value at point P and its maximum value at point Q . The values of the function lie within a range along the set of feasible solutions. To verify this, we place a point on the graph.

The geometric method of linear programming becomes more complex as the number of variables that enter the system of equations and the objective function increases. Therefore, if we have 2 or 3 variables, solving the problem graphically is feasible and appropriate.

We can also attempt to solve the problem of finite linear programming using the geometric method.

Example;

$$\begin{cases} 3x_1 + 4x_2 \leq 21/4 \\ 2x_1 + 5x_2 \leq 10 \\ x_1, x_2 \geq 0 \end{cases}$$

Find the derivative $F \max = 5x_1 + 11x_2$ of the function.



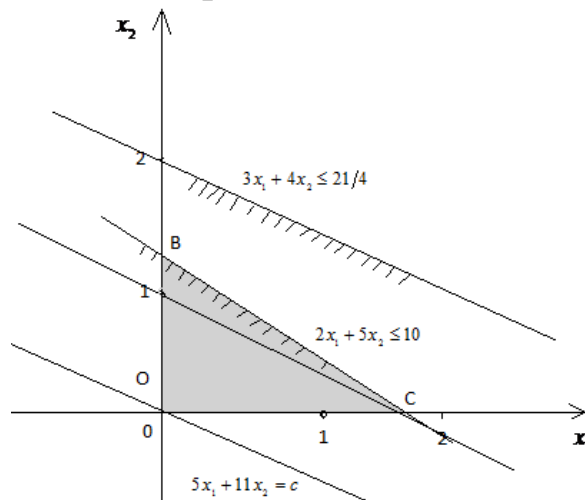
Solution. To solve the problem, let's illustrate the process of solving the problem graphically.

$$\begin{cases} 3x_1 + 4x_2 \leq 21/4 \\ 2x_1 + 5x_2 \leq 10 \\ x_1, x_2 \geq 0 \end{cases}$$

We solve the problem graphically.

We obtain the graph of the derivative, which represents the rate of change of the function.

We can obtain the derivative that represents the rate of change of the equation.



2-Figura

As you can see from the diagram, the solution polygon consists of OBC, and the problem consists of all solutions at points (1; 0) * and (0; 1) in the polygon. We put these points in the objective function (1; 0) at the point $F = 5x_1 + 11x_2 = 5 * 1 + 11 * 0 = 5$

$$F = 5x_1 + 11x_2 = 5 * 0 + 11 * 1 = 11$$

It can be seen that the objective function has a maximum value at the point (0 / (1 deg) F max = 11

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