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PERFORMING CLASSICAL ARITHMETIC OPERATIONS OF INTERVAL NUMBERS IN THE PYTHON PROGRAMMING LANGUAGE

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Abstract

There is the following definition of classical interval arithmetic [1]. Is given as the range of a real Arrow and its numbers, including the set of all real numbers that lie between themselves, i.e.,

There is the following definition of classical interval arithmetic: [1] the right R-axis interval is all the right numbers between these numbers, including themselves

$$a = \left[\underline{a}, \overline{a}\right] = \left\{x \in R \mid \underline{a} \le x \le \overline{a}\right\}. \quad (1.1)$$

Any interval has, in addition to its limits, the following. Features more commonly used in practice: $mid(a) = \frac{1}{2}\left(\overline{a} + \underline{a}\right) -$ middle of range (Center), $\rho(a, b) = \max\left\{\left|\underline{a} - \underline{b}\right|, \left|\overline{a} - \right.\right.$ $\operatorname{sign} a = \{$ istance between two interval numbers, $rad(a) = \frac{1}{2}\left(\overline{a} - \underline{a}\right) -$ the the interval is, $wid(a) = \overline{a} - \underline{a} -$ is the width of the interval and a the range is said to be broken, if $wid(a) = 0$ while, *is called degeneration*. In addition, there are the concepts of magnitude (modulus), interval momentum, relative width and others, as well as a number of properties inherent in these properties. $a, b \in$ for intervals, if a, $\underline{a} \le \underline{b}$ va $\overline{a} \le \overline{b}$ while, $b * (a - b)$ it is common to assume that it does not exceed. a an interval is non-negative if both ends are non-negative $(a \ge 0)$ is called. If both ends are not positive, a range is not positive $(a \le 0)$ is called [1].

$+, if \quad a \ge 0,$

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--, *if* $a \le 0$,
indefinite, *if* $0 \in a$

In general, an interval value (interval parameter) is said to be given if there is a variable that can take values in a given range. In mathematical language, an ordered pair is called an intermediate value, which can be defined by a special method $[\underline{a}, \overline{a}]$, where is the range of the variable and the values it can take. $a_1 \in a_1, a_2 \in a_2, \dots, a_n$ interval values are called free variables if the ordered set of corresponding variables $(a_1, a_2, \dots, a_n)$ if takes any values from their change $(a_1, a_2, \dots, a_n)$ is called the cartesian product of intervals [2].

The interval arithmetic operations of addition, subtraction, multiplication, and division are determined "by representatives", i.e., according to the following basic principle:

$$a \cdot b = \{a \cdot b \mid a \in \mathbf{a}, b \in \mathbf{b}\} \quad (1.2)$$

a, b for intervals, $a * b$, $* \in \{+, -, \cdot, /\}$ performing an asterisk operation will make sense $a \in [\underline{a}, \overline{a}]$, $b \in [\underline{b}, \overline{b}]$ for any. Where the real numbers a are defined by zero latitude intervals and are also called degenerate intervals.

Detailed definitions equal to (1.2) for interval arithmetic operations are given based on the following formulas.

$$a + b = [\underline{a} + \underline{b}, \overline{a} + \overline{b}], \quad (1.3)$$

$$a - b = [\underline{a} - \overline{b}, \overline{a} - \underline{b}], \quad (1.4)$$

$$a \cdot b = [\{\underline{a}\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}, \overline{a}\overline{b}\}; \max\{\underline{a}\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}, \overline{a}\overline{b}\}], \quad (1.5)$$

$$a/b = a \cdot [1/\overline{b}, 1/\underline{b}], \text{ here } b \not\ni 0. \quad (1.6)$$

Thus, the set of all intervals $a = [\underline{a}, \overline{a}] = \{x \in R \mid \underline{a} \le x \le \overline{a}\}$ (1.3)-(1.6) binary operations of addition, subtraction, multiplication, and division defined by formulas $(\square, +, -, *, /)$ *called classical arithmetic operations* [1].

It should be noted that, in general, interval division is not the inverse of addition, and interval division is not the inverse of multiplication. In addition, there is an important property of monotonicity: any interval and any $* \in \{+, -, \cdot, /\}$, operation

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$$a \subseteq b \Leftrightarrow \underline{a} \geq \underline{b} \quad \text{and} \quad \overline{a} \leq \overline{b},$$
$$c \subseteq d \Leftrightarrow \underline{c} \geq \underline{d} \quad \overline{c} \leq \overline{d},$$

and

$$a \subseteq c,\ b \subseteq d \Rightarrow \quad a \cdot b \subseteq c \cdot d.$$

Interval arithmetic operations have the following properties: associativity of addition, associativity of multiplication, commutativity of addition, commutativity of multiplication. A characteristic feature of interval arithmetic is that the product is not divided by addition: in general $(a+b)c \neq ac+bc$. Nevertheless, the weak property of the subdistributive of multiplication with respect to addition occurs: $a(b+c) \subseteq ab+ac$. In addition to arithmetic operations (1.3) - (1.6), interval analogs of elementary functions are used in interval calculations. If $r(x)$ □ at continuous unar is an operation, then defines the following operation that belongs to the real set

$$r(x) = \left[\min_{x \in x} r(x),\ \max_{x \in x} r(x)\right] \tag{1.7}$$

As examples of such unar operations $exp(x)$, $ln(x)$, $sin(x)$ and others can be cited: analytical expressions consisting of variable symbols, constants, four arithmetic operations (addition, subtraction, multiplication, division) and elementary functions, which we have called elementary functional expressions.

Solving any complex practical issue requires the execution of Whole chains of arithmetic operations.

As you know, $f : Rn \to R$ function if is an analytic expression, then $x_1, x_2, \ldots, x_n$ a finite combination of variables with four arithmetic operations and constants is called a rational function.

Theorem. $f(x_1,\ x_2,\ \ldots,\ x_n)$ of the rational function $(x_1,\ x_2,\ \ldots,\ x_n)$ for $F(X_1,\ X_2,\ \ldots,\ X_n)$ the result is determined $(X_1,\ X_2,\ \ldots,\ X_n) \in$ □ the change of its arguments and the execution of other operations act according to the rules of interval arithmetic [4].

Then

$$\{f(x_1, x_2, \ldots x_n) \mid x_1 \in x_1, x_2 \in x_2, \ldots, x_n\} \subseteq f(x_1, x_2, \ldots, x_n),$$

$F(X_1,\ X_2,\ \ldots,\ X_n)$ contained $f(x_1,\ x_2,\ \ldots,\ x_n)$ of the function $(X_1,\ X_2,\ \ldots,\ X_n)$ for arguments $f(x_1,\ x_2,\ \ldots,\ x_n)$ has an exact equality if the expression contains at most one representation of each variable in the first level.

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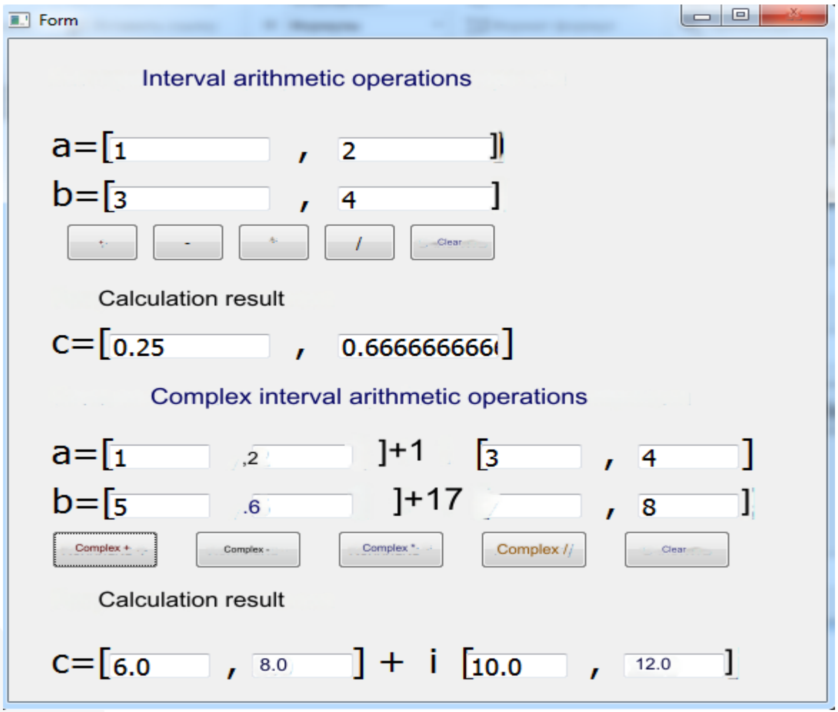
In the theory of interval analysis, the problem of interval estimation of the interval of values of functions based on two natural ideas is very important:
1) Their variability (1.1) is replaced by an analytical expression (or, instead of input arguments, a function is calculated and the usual arithmetic operations and elementary functions are replaced by their intervals (1.3) - (1.7), calculating the resulting interval expression after obtaining the result is performed, thus the interval obtained will have the interval of the desired values [6].
2) The original expression (or the algorithm for calculating the function) is replaced by another one that is equivalent in one way or another, but allows to be precise in the evaluation of the intervals, then the first idea is applied to the resulting expression.
The theoretical formulation of interval evaluation results is based on the following concepts. $f:\square^n \to \square^m$ the interval function is at this point $f(x)$ of the function $f:\square^n \to \square^m$ in the collection $D \subset \square^n$ If the duration of the range is for all point arguments $f(x) = f(x)$ $x \in D$ is the interval. $f:\square^n \to \square^m$ is called the interval extension of the point function $f:\square^n \to \square^m$ from $D \subset \square^n$ until, if 1) f of D has an interval extension to , 2) is monotone by adding to the identity, i.e $\forall x, y \in D$ for $x \subseteq y \Rightarrow f(x) \subseteq f(y)$ the implication is fulfilled. In addition, if f – if is an interval extension for a point function, then any point $x \in \square^n$ and any $x \in x$ for we have $f(x) = f(x) \in f(x)$, that is why $f(x)$ value x it is an outer interval, an estimate of the range of values of the function. An elementary extension of an elementary functional expression is obtained as a result of replacing its arguments with intervals of their change, replacing arithmetic operations and elementary functions with their interval analogues and extensions [2].
In addition to classical interval arithmetic, full interval arithmetic KR (Kaucher interval arithmetic) is distinguished, the description of its minimax characteristic and advantages are given in [3], [4]; complex interval arithmetic IC; twin arithmetic, originally introduced in the works of E. Gardenies and his colleagues [9]; Kahan interval arithmetic [5], in which a and b division of intervals in addition $0 \in b$ defined for , leading to infinite intervals; multi-interval arithmetic [5]; affine interval arithmetic [6], which differs from the classical one, and also allows taking into account the interrelationship between the quantities that appear in the calculation process; also called segment arithmetic.
Below we have created a visual program in the Python programming language for 4 operations related to classical interval arithmetic. We show here that the width of the confidence interval after the point increases compared to the results obtained in the

Intlab, Maple, InTAN, and Scilab packages and the C# and C++Builder programming languages [14], where we used previous intervalizers. We have shown it in our example.



In this figure, we have summarized interval arithmetic in the Python programming language. We can see the results of classical interval operations and complex interval 4 operations in one form. Interval Programming in languages close to classical interval subprogramming languages produces the desired results in most cases.

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