



## THEORETICAL FOUNDATIONS OF THE PROCESS OF PUNCH AND DIE CUTTING OF TEETH

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### Abstract

This article discusses the process of cutting teeth on saw blades in the interaction of the punch and the matrix. Modeling of the punching process analyzes the stress distribution in the contact area of the punch and the workpiece. Unlike traditional approaches using the Hertz model, a model of specific stress distribution is used for punching, taking into account the features of the punching process.

A system of equations is compiled, which is a generalized mathematical model of the cutting process, taking into account all the aspects considered earlier: stress distribution, fracture criterion, crack growth, process dynamics, tool wear, thermomechanical effects and the condition for the formation of a cut.

**Keywords:** Punch, matrix, punching process, mathematical model, stresses, dynamics, crack, thermomechanical effects.

### Introduction

In an era of rapid development of materials processing technologies, traditional approaches to the analysis of shaping processes require a fundamental rethinking. Teeth die-cutting of gin and linter saws is a unique case of localized high-speed interaction between tool and workpiece, where classical models of continuum mechanics face limitations due to the complexity of physical processes at the micro- and nanoscales[1-2].

The transition from the guillotine principle to die-cutting with the entire surface of the punch marks a new chapter in understanding the mechanics of material failure under impulse loads. This phenomenon requires the integration of knowledge from various fields of science, including solid state physics, materials science, and nonlinear.[3-4]





As part of this study, we propose an innovative concept for multiscale analysis of the die-cutting process, which allows us to overcome the limitations of existing models and open up new horizons in process optimization[1-5]. Our approach is based on the following key aspects:

1. Mesoscopic analysis of the evolution of the dislocation structure of the material in the deformation zone, taking into account the stochastic nature of the processes of defect self-organization.
2. Macroscopic modeling of wave processes in the punch-blank system taking into account nonlinear effects and localization of deformations.
3. A synergetic approach to the analysis of the interaction of various scale levels, which allows to identify the emergent properties of the system.
4. Development of a new theory of "dynamic surface morphology" describing the evolution of microrelief in the process of die-cutting and its effect on the functional properties of teeth.

Particular attention should be paid to the dynamic reconfiguration of the crystal lattice in the contact zone of the punch and the workpiece, which, according to our hypothesis, plays a key role in the formation of the unique properties of the tooth surface. This effect, previously not taken into account in classical models, can become the basis for the development of fundamentally new methods for controlling the properties of materials at the atomic level.

Modeling the felling process is a complex task that requires taking into account various physical phenomena and their interaction. The model is based on the analysis of the stress distribution in the contact area of the punch and the workpiece (Fig. 1).

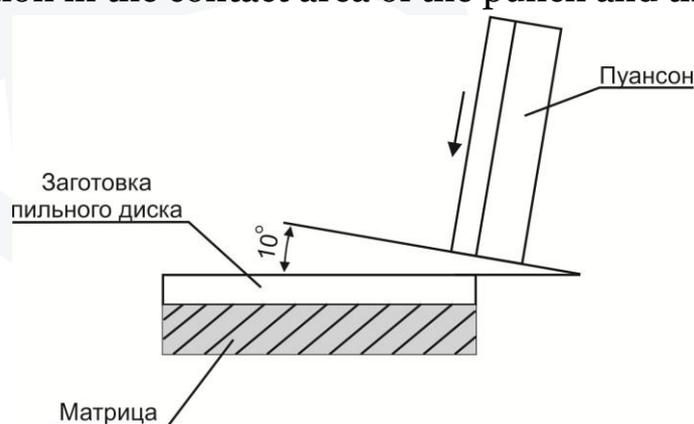


Figure 1. Diagram of the interaction between a punch and a matrix

In contrast to traditional approaches using the Hertzian model, it is more appropriate to use a specific stress distribution for cutting, taking into account the characteristics of the process:



$$\sigma(z) = k * \sigma_s * \left(1 - \frac{z}{s}\right) \quad (1)$$

Where:  $\sigma(z)$  is the stress at the distance  $z$  from the cutting edge,  $k$  is the coefficient depending on the properties of the material (usually taking values of 1.1-1.3),  $\sigma_s$  is the shear resistance of the material, and  $s$  is the thickness of the material.

The cutting process is characterized by a complex stress-strain state that cannot be adequately described by the classical Mises criterion. Instead, it is proposed to use a combined failure criterion that takes into account both equivalent and shear stresses:

$$\left(\frac{\sigma_{eq}}{\sigma_y}\right)^2 + 2\left(\frac{\tau_{max}}{\tau_c}\right)^2 = 1 \quad (2)$$

where  $\sigma_{eq}$  is the equivalent voltage,

- yield strength,  $\sigma_y$

$\tau_{max}$  is the maximum shear stress,

- Shear strength.  $\tau_c$

Note that  $\tau_{max}$  can be expressed in terms of  $\sigma(z)$  from equation (1), establishing a relationship between the stress distribution and the failure criterion.

To describe the growth of a fracture during the cutting process, an effective approach is to use a cohesive zone model that relates the stress vector in the fracture zone to the fracture opening vector:

$$t = f(\delta) \quad (3)$$

where  $t$  is the stress vector in the cohesive zone,  $\delta$  is the fracture opening vector, and  $f$  is a function describing the relationship between stress and opening. The function  $f$  can be determined experimentally or on the basis of theoretical models that take into account the properties of the material.  $f(\delta)$

The dynamics of the cutting process is described by the equation of motion:

$$\rho \frac{\partial^2 u}{\partial t^2} = \nabla \cdot \sigma + f \quad (4)$$

where  $\rho$  is the density of the material,  $u$  is the displacement vector,  $\sigma$  is the stress tensor, and  $f$  is the vector of volume forces.

The stress tensor  $\sigma$  in this equation is related to the stress distribution described by equation (1).

When modeling the punching process, it is important to take into account tool wear, which can be described using a modified Archard's law:

$$V = k * F_N * \frac{s}{H} \quad (5)$$

where  $V$  is the volume of worn material,  $k$  is the wear coefficient,  $F_N$  is the normal force,  $s$  is the sliding path,  $H$  is the hardness of the tool material.  $F_N$



$F_N$  here can be expressed in terms of the integral of  $\sigma(z)$  from equation (1) over the contact area.

Thermomechanical coupling in the felling process is described by a system of equations:

$$\sigma = \frac{f(\varepsilon, \dot{\varepsilon}, T)(6) \partial T}{\partial t} = \alpha \nabla^2 T + \left( \frac{\beta}{\rho c_p} \right) * \sigma : \dot{\varepsilon}_p (7)$$

where  $\varepsilon$  is the deformation,  $\dot{\varepsilon}$  is the strain rate,  $T$  is the temperature,  $\alpha$  is the thermal diffusivity coefficient,  $\beta$  is the Taylor-Queenie coefficient  $c_p$  is the specific heat capacity.

Equation (6) relates stresses to strains and temperature, and equation (7) describes the temperature change during the deformation process.

The key condition for the formation of a cut during punching is that the integral value of the shear stresses along the thickness of the material reaches the critical value:

$$\int_0^S \tau(z) dz = \sigma_s * s \quad (8)$$

where  $\tau(z)$  is the shear stress at the distance  $z$  from the cutting edge.

This condition is directly related to the stress distribution described by equation (1), since  $\tau(z)$  can be expressed in terms of  $\sigma(z)$ .

To fully describe the punching process, it is necessary to take into account the boundary conditions, including the movement of the punch, the applied force and the conditions for fixing the workpiece. These conditions affect the solution of the equation of motion (4) and determine the specific values in equations (1)-(3) and (5)-(8).

Summarizing all of the above, it is possible to present a complex model of the felling process in the form of a system of equations:

$$\left\{ \begin{array}{l} \sigma(z) = k * \sigma_s * \left( 1 - \frac{z}{s} \right) \\ \left( \frac{\sigma_{eq}}{\sigma_y} \right)^2 + 2 \left( \frac{\tau_{max}}{\tau_c} \right)^2 = 1 \\ t = f(\delta) \\ \rho \frac{\partial^2 u}{\partial t^2} = \nabla \cdot \sigma + f \\ V = k * F_N * \frac{s}{H} \\ \sigma = f(\varepsilon, \dot{\varepsilon}, T) \\ \frac{\partial T}{\partial t} = \alpha \nabla^2 T + \left( \frac{\beta}{\rho c_p} \right) * \sigma : \dot{\varepsilon}_p \\ \int_0^S \tau(z) dz = \sigma_s * s \end{array} \right. \quad (9)$$



This system of equations (9) is a generalized mathematical model of the punching process, taking into account all the aspects considered earlier: stress distribution (the first equation, corresponds to (1)), the fracture criterion (the second equation, corresponds to (2)), crack growth (the third equation, corresponds to (3)), the dynamics of the process (the fourth equation, corresponds to (4)), tool wear (the fifth equation, corresponds to (5)), thermomechanical effects (the sixth and seventh equations, correspond to (6) and (7)) and the condition for the formation of the slice (the eighth equation, corresponds to (8)).

The solution of this system of equations, taking into account the corresponding boundary conditions, makes it possible to obtain a complete description of the cutting process and predict the quality of the resulting parts. It is important to note that the equations in system (9) are interrelated: the stress distribution affects the fracture criterion, crack growth, process dynamics, tool wear, thermomechanical effects, and slice condition. This makes the system nonlinear and requires the use of complex numerical methods to solve it.

Based on the equation described above, to determine the force, you can use the classical formula for calculating the punching force at zero clearance, widely used in engineering practice, has the form:

$$F_0 = \sigma_s * L * \frac{s}{2}$$

where  $F_0$  is the punching force at zero gap,  $\sigma_s$  is the shear strength of the material,  $L$  is the perimeter of the cut contour,  $s$  is the thickness of the material [6-8]. However, this formula does not take into account the effect of the gap between the punch and the die, which significantly affects the punching process.

To take into account the effect of the gap on the punching force, a theoretical model based on the analysis of the stress distribution in the shear zone is proposed. According to the theory of plasticity [6-8], the distribution of stresses in the shear zone can be approximated by a linear function:

$$\sigma(x) = \sigma_s * \left(1 - \frac{x}{s_{eff}}\right) \quad (10)$$

where  $\sigma(x)$  is the stress at a distance  $x$  from the cutting edge,  
- effective cut thickness.  $s_{eff}$

The  $z$  gap between the punch and the die affects the effective slice thickness, which can be expressed as:

$$s_{eff} = s + k * z \quad (11)$$

where  $k$  is a coefficient that takes into account the geometry of the deformation.



To obtain the punching force, we need to integrate the stresses over the effective cutting area. Let's start with the expression for stress distribution (10) and integrate it over the effective slice thickness:

$$F = \int_0^{s_{eff}} \sigma(x) * L dx$$

where L is the perimeter of the cut contour.

Substituting the expression for from formula (10), we get: $\sigma(x)$

$$F = \int_0^{s_{eff}} \sigma_s * \left(1 - \frac{x}{s_{eff}}\right) * L dx$$

Calculate the integral:

$$F = \sigma_s * L * \left[ x - \frac{x^2}{2*s_{eff}} \right]_{0}^{s_{eff}}$$

$$F = \sigma_s * L * \left[ s_{eff} - \frac{s_{eff}^2}{2*s_{eff}} \right]$$

$$F = \sigma_s * L * \frac{s_{eff}}{2}$$

Now substitute the expression for from formula (11): $s_{eff}$

$$F = \sigma_s * L * \frac{(s + k * z)}{2}$$

Let us express the gap z as a fraction of the thickness of the material:  $z = \varepsilon * s$ , where  $\varepsilon$  is the relative clearance. Substituting in the previous formula we get:

By integrating stresses over the effective cutting area and expressing the gap z as a fraction of the thickness of the material ( $z = \varepsilon * s$ ), we obtain the dependence of the punching force on the relative clearance:

$$F(\varepsilon) = F_0 * (1 + k * \varepsilon) \quad (12)$$

This formula shows that the punching force increases linearly with increasing clearance, and the rate of this increase is determined by the coefficient k.

For the analytical derivation of the coefficient k, let's consider the geometry of the material deformation in the shear zone. Suppose that the deformation of the material occurs in the triangular zone between the cutting edges of the punch and the die. Let  $\alpha$  be the angle of inclination of the cut line to the vertical, then:

$$\tan(\alpha) = \frac{z}{s} = \varepsilon \quad (13)$$

The effective thickness of the cut will be equal to the length of the inclined cut line: $s_{eff}$

$$s_{eff} = \frac{s}{\cos(\alpha)} \quad (14)$$

By decomposing  $\cos(\alpha)$  into a Taylor series to the second order, and taking into account that  $\tan(\alpha) \approx \alpha$  for small angles, we get:

$$s_{eff} \approx s * \left(1 + \frac{\varepsilon^2}{2}\right) \quad (15)$$



Comparing this expression with the previously introduced formula (11) for  $k$ , we see that the coefficient  $k$  in the first approximation is equal to half of the relative gap:  $s_{eff}$

$$k \approx \varepsilon / 2 \quad (16)$$

Thus, the final formula for the clearance-taking force takes the form for the normal case:

$$F(\varepsilon) \approx F_0 * \left(1 + \frac{\varepsilon^2}{2}\right) \quad (17)$$

And if the gap increases, then we will use equation (12). This dependence shows that the effect of the gap on the cutting force is non-linear. At small values of the clearance ( $\varepsilon < 0.1$ ), the increase in the cutting force is insignificant, which is consistent with the experimental data given in the work of M.E. Zubtsov. [6]. However, in real-world production conditions, the gap between the punch and the die can vary more widely. In addition, as the gap increases, the punching process becomes more complex, including not only the cut but also the significant bending of the material.

Taking into account these factors, it becomes obvious that the simple linear dependence of the punching force on the gap, expressed by equation (12), cannot adequately describe the process in the entire range of possible clearance values. Moreover, in the previous analysis, we did not take into account the influence of strain rate and dynamic effects, which can play a significant role in high-speed cutting.

In order to more accurately describe the punching process under different conditions, it is necessary to develop a more complex model that would take into account not only the size of the gap, but also the properties of the material, the geometry of the process and the rate of deformation. Of particular interest is the coefficient  $k$  in equation (12), which, as we now understand, cannot be constant, but must depend on many factors. In this regard, it is proposed to conduct a more in-depth analysis based on the energy approach using Lagrange equations. This method will allow us to take into account the kinetic and potential energy of the system, as well as the dissipation energy associated with plastic deformation and cracking.

Let's start our analysis by considering the energy balance of the felling process. The total energy of the system  $E$  can be represented as the sum of the kinetic energy  $T$ , the potential energy  $U$  and the dissipation energy  $D$ :

The total energy of the system  $E$  can be represented as the sum of the kinetic energy  $T$ , the potential energy  $U$  and the dissipation energy  $D$ :

$$E = T + U + D \quad (18)$$

The kinetic energy  $T$  depends on the velocity of the punch  $v$ :

$$T = \left(\frac{1}{2}\right) * m * v^2 \quad (19)$$



where  $m$  is the effective mass of the system.

The potential energy  $U$  includes the strain energy of the material during bending and tension (20):  $U = U_{\text{изгиб}} + U_{\text{растяж}}$

The bending energy can be approximated as:

$$U_{\text{изгиб}} = \left(\frac{1}{2}\right) * EI * \left(\frac{\varepsilon}{s}\right)^2 * L \quad (21)$$

where  $E$  is Young's modulus,  $I$  is the moment of inertia of the section,  $\varepsilon$  is the relative clearance,  $s$  is the thickness of the material,  $L$  is the perimeter of the notching.

Tensile Energy:

$$U_{\text{растяж}} = \left(\frac{1}{2}\right) * EA * \left(\frac{\Delta l}{l}\right)^2 * l \quad (22)$$

where  $A$  is the cross-sectional area,  $\Delta l$  is the elongation, and  $l$  is the characteristic length.

The dissipation energy  $D$  takes into account plastic deformation and crack formation:

$$D = \sigma_y * V_{\text{пласт}} + G_c * A_{\text{трещ}} \quad (23)$$

where  $\sigma_y$  is the yield strength,  $V_{\text{пласт}}$  is the volume of the plastically deformed material,  $G_c$  is the critical fracture energy,  $A_{\text{трещ}}$  is the area of the crack formed.

Applying the Lagrange equation:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = Q \quad (24)$$

where  $L$  is the Lagrange function,  $L = T - U$

$q$  is the generalized coordinate (in our case, the displacement of the punch),  $Q$  is the generalized force.

By substituting expressions for  $T$  and  $U$  and performing differentiation, we get:

$$m * \frac{d^2 q}{dt^2} + \left( \frac{EI}{s^2} + \frac{EA}{l} \right) * q = F - \frac{\partial D}{\partial q} \quad (25)$$

where  $F$  is the applied force.

Given the rate of deformation, we can express  $F$  as:  $\dot{\varepsilon} = \frac{v}{s}$

$$F = F_0 * (1 + k\varepsilon) * (1 + \eta\dot{\varepsilon}) \quad (26)$$

where  $\eta$  is the coefficient of sensitivity to the strain rate.

Now, comparing (25) and (26), we can express the coefficient  $k$ :

$$k = \left[ \frac{\left( \frac{EI}{s^2} + \frac{EA}{l} \right)}{F_0} + \frac{\partial D}{\partial q F_0} \right] * s - \frac{\eta v}{s} \quad (27)$$

This expression shows that  $k$  depends on:

1. Material properties ( $E, \sigma_y, G_c$ )
2. Process geometries ( $I, A, s, l$ )
3. Deformation Rates ( $v$ )
4. Clearance ( $\varepsilon$ )



## Conclusion

For the practical use of this formula, it is necessary to determine the values of the parameters included in it experimentally or from reference data for a specific material and process conditions.

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