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STATISTICAL ANALYSIS OF SATURATION THICKNESS IN BREMSSTRAHLUNG GAMMA-RAY SCATTERING FROM LEAD TARGETS

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Abstract

The present paper considers a statistical analysis of the dependence of the saturation thickness of scattered gamma radiation on the detection angles and the orientation of flat lead targets relative to the direction of the probing beam. The analysis was performed using the least-squares method. Experimental data obtained from the scattering spectra of bremsstrahlung gamma radiation were processed to determine the influence of target orientation and detection geometry on the saturation thickness of scattered radiation.

Keywords: Scattering spectra, gamma radiation, target orientation, distribution, probing beam, least-squares method, saturation thickness.

Introduction

A large number of investigations employing radionuclide sources (at the early stages of research) and bremsstrahlung radiation generated by accelerated electrons with energies in the range of 10–900 MeV have established the fundamental regularities governing the formation of gamma-ray backscattering fields (gamma-ray albedo) from various targets with effective atomic numbers in the range Z_eff = 5–82.

These studies demonstrated the possibility of determining the effective atomic number, dimensions, orientation, density distribution within the volume of the material, and other characteristics of investigated targets that are important for solving a wide range of fundamental and applied scientific problems.

In contrast, the scattering of gamma radiation into the forward hemisphere along the direction of the probing beam has been investigated only recently because of the high background level in this direction. Such investigations have mainly been carried out in our laboratory.

The conducted studies have shown that:

- The spectra of scattered gamma radiation consist of monochromatic isotropic components (characteristic X-ray and annihilation radiation) and continuous anisotropic components (Compton scattering, bremsstrahlung radiation of secondary electrons and positrons, and background radiation), whose relative intensities depend on the effective atomic number Z_eff and target thickness d.
- The backscattered gamma-radiation method is applicable for investigating objects with thicknesses smaller than the saturation thickness d_0. The forward-scattering method does not have such limitations. When $d > d_0$, the yield of backscattered radiation remains practically constant because the radiation generated in deeper layers is absorbed before leaving the object. In contrast, the yield of forward-scattered radiation decreases because the increase in absorption exceeds the increase in generation.
- The saturation thickness d_0 depends on the effective atomic number of the target material and the maximum energy E_e of the probing radiation.

It is evident that the saturation thickness d_0 should also depend on the scattering (detection) angle θs and the orientation angle φ of the target relative to the direction of the probing beam. The investigation of these dependences constitutes the main objective of the present work.

2. Numerical Characteristics of the Investigated Scattered Gamma-Ray Fluxes

Processing of the experimental spectra made it possible to determine the numerical characteristics necessary for analyzing the formation mechanisms of scattered gamma-radiation fields.

Table 1. Relative Intensity of the Annihilation Component NA (0.45–0.55 MeV) in Bremsstrahlung Scattering Spectra

d, mm	θ_s=10°		
	φ=45°	φ=90°	φ=135°
5	19,7	26,8 (22,4)	21,8
10	29,5	33,5 (28,6)	34,7
15	31,6	41,0 (34,9)	41,0
20	27,1	43,5 (36,7)	36,0
25	22,6	38,2 (33,3)	32,4
30	16,5	35,1 (31,4)	25,0
35	14,3	29,2 (26,5)	20,9
40	11,1	26,5 (24,1)	15,8
D_0, mm	14,6	18,7 (19,9)	15,1

Statistical Analysis. Time Series and Their Characteristics

The study of patterns in the temporal variation of events is one of the fundamental issues of statistics.

Such problems are generally addressed through the construction and analysis of time series.

A sequence of statistical observations recorded at a specific moment or over a certain period of time constitutes a time series.

The statistical values that form a time series are referred to as series levels. In most cases, time series are presented in the form of tables or graphs.

Time-series analysis involves solving the following tasks:

1. Identifying the main factors that influence changes in the levels of the series (smoothing or trend analysis);
2. Forecasting future levels of the series and estimating values for subsequent time periods;
3. Interpolation, i.e., determining unknown series levels for intermediate time points based on neighboring observed values;
4. Establishing functional relationships between the levels of one or more time series;
5. Interpreting and analyzing periodic fluctuations within the series.

In the present study, attention is focused primarily on Tasks 1 and 2, which are considered the most important aspects of time-series analysis [4].

Determination of the Main Factors Influencing Changes in Series Levels (Trend Analysis)

Table 2. Numerical Characteristics of Scattered Gamma Radiation (SGR) Fluxes

The scattering spectra of bremsstrahlung radiation generated by electrons with energies of **13 MeV** and **22.5 MeV** from lead scatterers with thicknesses **d = 5–40 mm**, measured at the scattering angle **θs = 10°** and target orientation **φ = 45°**, are represented by the series Xi(t)X_i(t)Xi(t) as follows:

Table 2.

(i) d (mm)	5	10	15	20	25	30	35	40	D₀, мм
X_i (t) $\theta_s=10°$; $\varphi=45°$	19,7	29,5	31,6	27,1	22,6	16,5	14,3	11,1	14,6

a = $\overline{X}$ = (1/n) $\sum_{i=1}^{n} X_i$ = 172,4/8=21,55.

$b = \frac{\sum_{i=1}^{n} X_i \cdot t_i'}{\sum_{i=1}^{n} (t_i')^2}$; $b = \frac{\sum_{i=1}^{n} X_i \cdot t_i'}{\sum_{i=1}^{n} (t_i')^2} = \frac{-551,2}{1052} \approx - 0,52$.

$\overline{X}(t_i') = a + b(t_i') = 21,55 - 0,52t_i'$.

$t_1' = - 18$; $\overline{X}(-18) = 21,55 + (-0,52)(-18) = 21,55 + 9,36 = 30,91$.
$t_2' = - 13$; $\overline{X}(-13) = 21,55 + (-0,52)(-13) = 21,55 + 6,76 = 28,31$.
$t_3' = - 8$; $\overline{X}(-8) = 21,55 + (-0,52)(-8) = 21,55 + 4,16 = 25,71$.
$t_4' = - 3$; $\overline{X}(-3) = 21,55 + (-0,52)(-3) = 21,55 + 1,56 = 23,11$.
$t_5' = 2$; $\overline{X}(2) = 21,55 + (-0,52)(2) = 21,55 - 1,04 = 20,51$.
$t_6' = 7$; $\overline{X}(7) = 21,55 + (-0,52)(7) = 21,55 - 3,64 = 17,91$.
$t_7' = 12$; $\overline{X}(12) = 21,55 + (-0,52)(12) = 21,55 - 6,24 = 15,31$.
$t_8' = 17$; $\overline{X}(17) = 21,55 + (-0,52)(17) = 21,55 - 8,84 = 12,71$.

$\overline{X}(t_i') = a + b(t_i') + c(t_i')^2 = 25,11 - 0,52(t_i') - 0,027(t_i')^2$.

$t_1' = - 18$; $\overline{X}(-18) = 25,11 - 0,52(-18) - 0,027(-18)^2 = 25,11 + 9,36 - 8,75 = 25,72$.
$t_2' = - 13$; $\overline{X}(-13) = 25,11 - 0,52(-13) - 0,027(-13)^2 = 25,11 + 6,76 - 4,56 = 27,31$.
$t_3' = - 8$; $\overline{X}(-8) = 225,11 - 0,52(-8) - 0,027(-8)^2 = 25,11 + 4,16 - 1,73 = 27,54$.
$t_4' = - 3$; $\overline{X}(-3) = 25,11 - 0,52(-3) - 0,027(-3)^2 = 25,11 + 1,56 - 0,24 = 26,43$.
$t_5' = 2$; $\overline{X}(2) = 25,11 - 0,52(2) - 0,027(2)^2 = 25,11 - 1,04 - 0,11 = 23,96$.
$t_6' = 7$; $\overline{X}(7) = 25,11 - 0,52(7) - 0,027(7)^2 = 25,11 - 3,64 - 1,32 = 20,15$.
$t_7' = 12$; $\overline{X}(12) = 25,11 - 0,52(12) - 0,027(12)^2 = 25,11 - 6,24 - 3,89 = 14,98$.
$t_8' = 17$; $\overline{X}(17) = 25,11 - 0,52(17) - 0,027(17)^2 = 25,11 - 8,84 - 7,80 = 8,47$.

Conclusion

Thus, the conducted investigations allow the following conclusions to be drawn:

- A variation in the probing beam energy within the range of 13–22 MeV affects the absolute value of the annihilation component intensity IA, while practically preserving the overall character of its dependence on the target thickness d. The quadratic-linear model provides the best approximation of the relationship between the absolute value of IA and the target thickness d.
- A variation in the probing beam energy within the range of 13–22 MeV also affects the absolute value of the Compton component intensity IK, while causing virtually

no change in the general character of its dependence on the target thickness d. The quadratic-linear model most accurately describes the dependence of the absolute value of IK on the target thickness d.

Therefore, among the considered approximation methods, the quadratic-linear model offers the most adequate representation of the experimental dependence of scattered gamma-radiation characteristics on the thickness of lead scatterers.

References

1. Pruitt, J.S. High-Energy X-Ray Photo Albedo. Nuclear Instruments and Methods, 1964, Vol. 27, No. 1, pp. 23–28.
2. Bulatov, B.P., Efimenko, B.A., Zolotukhin, V.G., Klimanov, V.A., and Mashkovich, V.R. Gamma-Ray Albedo. Moscow: Atomizdat Publishers, 1966.
3. Eager, R.G. (Ed.). Engineering Compendium on Radiation Shielding. Berlin, Heidelberg, New York: Springer-Verlag, 1968.
4. Ulugmurodov, N.X. Course of Mathematical Statistics: Textbook. Tashkent: TURON-IQBOL Publishing House, 2006, 207 p.
5. Bulatov, B.P., and Andryushin, N.F. Backscattered Gamma Radiation in Radiation Engineering. Moscow: Atomizdat Publishers, 1971.
6. Kumakhov, M.A. “On the Theory of Electromagnetic Radiation of Charged Particles in a Crystal.” Physics Letters A, Vol. 57, 1976, p. 17.
7. Golikov, E.G., Kovyazin, Yu.A., and Kortov, V.S. “Evaluation of the Informativeness of Scattered Radiation Characteristics from the Surface Area of Geometric Bodies.” Defektoskopiya, No. 11, 1982, pp. 86–92.
8. Vnukov, I.E., Vorobyev, S.A., et al. “Experimental Investigations of Gamma-Ray Albedo Generated by Ultrarelativistic Electrons.” Proceedings of Higher Educational Institutions of the USSR. Physics Series, No. 6, 1991, p. 106.
9. Alimov, G.R., Kungurov, F.R., Salikhbaev, U.S., Samatov, Zh.K., Safarov, A.N., Usmanov, R., Muminov, T.M., Mukhamedov, A., and Khazratov, T. “Investigation of Bremsstrahlung Gamma-Ray Scattering from Various Objects.” Bulletin of the Russian Academy of Sciences, Physics Series, Vol. 62, No. 5, 1998, p. 1072.
10. Alimov, G.R., Asatov, U.T., Kumakhov, M.A., Muminov, A.T., Muminov, T.M., Salikhbaev, U.S., Safarov, A.N., Usmanov, R.R., Khazratov, T., and Khakberdiev, I. “Scattering of Bremsstrahlung Radiation with a Maximum Energy of 22 MeV from Flat Scatterers.” Bulletin of the Russian Academy of Sciences, Physics Series, Vol. 64, No. 5, 2000, p. 1007.