WEB OF SCIENTIST: INTERNATIONAL SCIENTIFIC RESEARCH JOURNAL
ISSN: 2776-0979, Volume 3, Issue 1, Jan., 2022

THERMODYNAMIC PROPERTIES OF HORAVA-LIFSHITZ BLACK HOLE

A.I. Mamadjanov
Department of Physics, Namangan engineering-construction institute,
Namangan, Uzbekistan. Email:mamdjanov3084@gmail.com

O.R. Isoqova
Magister, Namangan state university, Uzbekistan,
Namangan, Uychi street 316

Annotation

The thermodynamic properties of the Hořava-Lifshitz black hole are investigated with new and simple methods. Hawking radiation and the life time of Horava-Lifshitz black hole has been studied. An exact analytical expression for the Bekenstein-Hawking's entropy, the temperature and specific heat capacity have been found. Time dependence of thermodynamic quantities of black hole has been studied

Keywords: Hawking radiation, Lifetime, Entropy, Hořava-Lifshitz black hole

Introduction

Combining the Einstein's theory of relativity with quantum mechanics or obtaining a theory of quantum gravity (QG) remains an unsolved problem in physics. Several attempts have been made in this regard [1]. In 2009 Peter Hořava proposed his theory for QG which is based on the explicit violation of local Lorentz invariance [2]. This model is, higher order gravity model and degenerates into the General Relativity (GR) at large distances.

Nowadays the Hořava-Lifshitz gravity theory has been widely investigated. For example, [3] the authors investigated the Dirac quasinormal modes of the black hole. And in [4] authors studied the Quantum Gravity effects on HLG model. Some static spherically symmetric black hole solutions have been obtained in the HL gravity theory [5]-[7]. Also 2D black holes with vanishing horizon area are discussed in references. [8]-[11].

Thermodynamics of different black holes with hyperscaling violation are studied in the references [10]-[11]. Recently the stability and Hawking-Page phase transition of the massive black holes with energy dependent spacetime are studied [12]-[14].

In this paper, we have studied the Hawking radiation and life time as well as the thermodynamic properties of the Hořava-Lifshitz black hole with simple methods.

The paper is organized as follows. Section 2 is devoted to the study of the Hawking radiation and life time of Hořava-Lifshitz Black hole. In section 3, we present the basic equations for the description of the black hole thermodynamics in the Hořava-Lifshitz garaity. The summary of the obtained results is reported in section 4.
Throughout the paper we employ the convention of a metric with signature (-,+,+,+). We use units in which $G=c=\hbar=k=1$, but we restore then when we have to compare our findings with observational date.

Hawking Radiation and Life Time of Hořava-Lifshitz Black Hole

In this section we study Hawking radiation and life time Hořava-Lifshitz black hole and evaluate the effect of the ω free parameter on the life time of the Horava-Lifshitz black hole. Before studying Hawking radiation, we need to determine the temperature of the black hole. In the spherical coordinates (t,r,θ,φ) the spacetime metric for a spherical symmetric, static Hořava-Lifshitz black hole is given by [15]

$$ds^2=-f(r)dt^2+\frac{1}{f(r)}dr^2+r^2(d\theta^2+\sin\theta^2 d\varphi^2) \quad (1)$$

with

$$f(r)=1+\omega r^2\left[1-\sqrt{1+\frac{4M}{\omega r^3}}\right], \quad (2)$$

Where M is the total mass of black hole and ω is a free parameter that regulates the ultraviolet behavior of the theory.
When taking into account the quantum properties of light (which were so far ignored), Hawking discovered in 1974 that newly formed black holes are not black [16]. Indeed, he found that they spontaneously emit a steady thermal flux of radiation at a temperature given by

$$k_B T_H=\hbar\omega \quad (3)$$

and he showed that temperature is would be

$$T_H=\frac{\hbar c^3}{8\pi k_B GM} \quad (4)$$

But the equation (4) does not determine the relationship between temperature and another parameter of black holes. Therefore, we use from the temperature of the black hole (Hawking temperature) which can be evaluated via the surface gravity method at the horizon for determined the relationship between temperature and ω free parameter of the Hořava-Lifshitz black hole:

$$T_H=\frac{\kappa_G}{2\pi}, \qquad \kappa_G=\frac{1}{2}\frac{dg_{tt}}{dr}\bigg|_{r=r_+} \quad (5)$$

where κ_G is surface gravity factor and r_+ is the radius of the outer horizon of the black hole. The radius of an outer horizon of Hořava-Lifshitz black hole can be calculated by condition, i.e. $f(r)=0$, that is

$$r_{+}=M\left[1+\sqrt{1-\frac{1}{2\omega M^{2}}}\right]\leq 2M, \tag{6}$$

which is further than Schwarzschild radius. Recalling equation (5) Hawking temperature over the surface of the Hořava-Lifshitz black hole can be

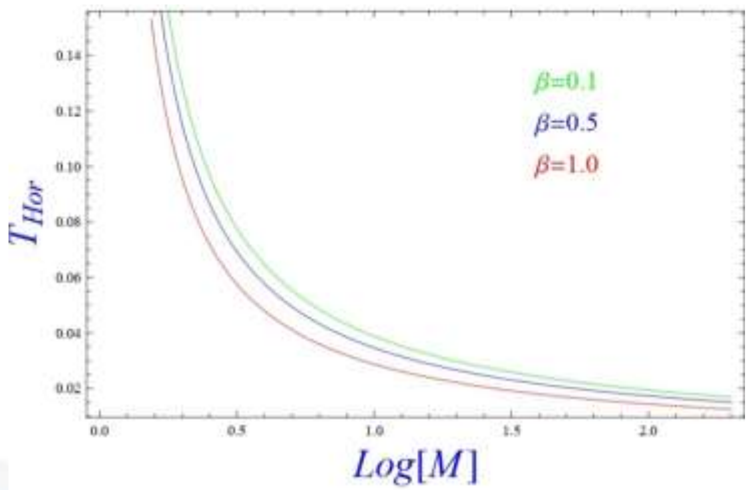

Figure 1.The dependence of the temperature of the Hořava-Lifshitz black hole on its mass, for different values of the parameter $\beta=1/\omega M^{2}$.

$$T_{H}=\frac{1}{2\pi M}\frac{\Lambda}{\beta}\left[1+\left(1-\frac{8}{\beta}\right)\left(\sqrt{1+32\frac{\Lambda}{\beta^{2}}+\frac{8}{\beta}\sqrt{1-\frac{\beta}{2}}}\right)^{-1}\right]\geq\frac{1}{8\pi M}, \tag{7}$$

where we used following notation

$$\Lambda=1+\sqrt{1-\frac{\beta}{2}},\qquad \beta=\frac{1}{\omega M^{2}}, \tag{8}$$

In fig.1 shown that the dependent of the temperature on its mass. Hawking radiation for a black hole is follows from Stephan-Boltzmann's law for the thermal radiation of absolute black body, i.e.

$$\frac{dE}{dt}\simeq\sigma AT^{4}=4\pi r_{+}^{2}\sigma T^{4} \tag{9}$$

where $\sigma=\pi^{2}/60$ is the Stephan-Boltzmann constant, A is the surface area of black hole horizon. Given that $E=M\cdot c^{2}$ and using equations (6,7) we can easily find following, i.e.

$$\frac{dM}{dt}=-\frac{1}{240\pi M^{2}}\frac{\Lambda^{6}}{\beta^{4}}\left[1+\left(1-\frac{8}{\beta}\right)\left(\sqrt{1+32\frac{\Lambda}{\beta^{2}}+\frac{8}{\beta}\sqrt{1-\frac{\beta}{2}}}\right)^{-1}\right]^{4} \tag{10}$$

From equation (10), we can determine the relationship between life time and free parameter of black hole:

$$\tau = -240\pi \int_M^0 M^2 \frac{\beta^4}{\Lambda^6}\left[1+\left(1-\frac{8}{\beta}\right)\left(\sqrt{1+32\frac{\Lambda}{\beta^2}+\frac{8}{\beta}\sqrt{1-\frac{\beta}{2}}}\right)^{-1}\right]^{-4} dM \qquad (11)$$

by assuming that $\beta = 1/\omega M^2 = const$ is constant, we can determine the lifetime of the Hořava-Lifshitz black hole, by calculating the integral (11).

$$\tau = 240\pi M^3 \frac{\beta^4}{\Lambda^6}\left[1+\left(1-\frac{8}{\beta}\right)\left(\sqrt{1+32\frac{\Lambda}{\beta^2}+\frac{8}{\beta}\sqrt{1-\frac{\beta}{2}}}\right)^{-1}\right]^{-4} \qquad (12)$$

where we used following notations

$$\Lambda = 1+\sqrt{1-\frac{\beta}{2}}, \qquad \beta = \frac{1}{\omega M^2}, \qquad (13)$$

In fig. 2 shows the graph of the life time of the Hořava-Lifshitz black hole for various values of the parameter $\beta = 1/\omega M^2$, depending on its mass.Note that case the general relativistic limit of life time takes the standard value as obtain in ref [16]

$$\tau_0 = \lim_{\omega\to\infty} \tau = 5120\pi M^3 \qquad (14)$$

For small parameter $\beta = 1/\omega M^2$ of life time of Hořava-Lifshitz black hole takes as aform

$$\tau = \tau_0\left(1+12\frac{1}{2}\beta+\frac{65}{64}\beta^2+\frac{179}{256}\beta^3\right)+o(\beta)^4 \qquad (15)$$

which shows that the life time of Hořava-Lifshitz black hole will be shorter with increasing of the parameter ($\beta = 1/\omega M^2$).

Thermodynamic Properties of Hořava-Lifshitz Black Hole

In this section we briefly study thermodynamic properties of the Hořava-Lifshitz black hole. The surface of an outer horizon can be found as $A = 4\pi r_+^2$. Once we have the surface of the horizon then area entropy of black hole can be easily calculated as

$$S = \frac{A}{4\pi} = \pi M^2\left[1+\sqrt{1-\frac{1}{2\omega M^2}}\right]^2 \le 4\pi M^2, \qquad (16)$$

According to the definition of the value of the free energy $F = M - TS$ of the thermodynamic system one can have

$$F = M - \frac{M}{2}\frac{\Lambda^3}{\beta}\left[1+\left(1-\frac{8}{\beta}\right)\left(\sqrt{1+32\frac{\Lambda}{\beta^2}+\frac{8}{\beta}\sqrt{1-\frac{\beta}{2}}}\right)^{-1}\right], \qquad (17)$$

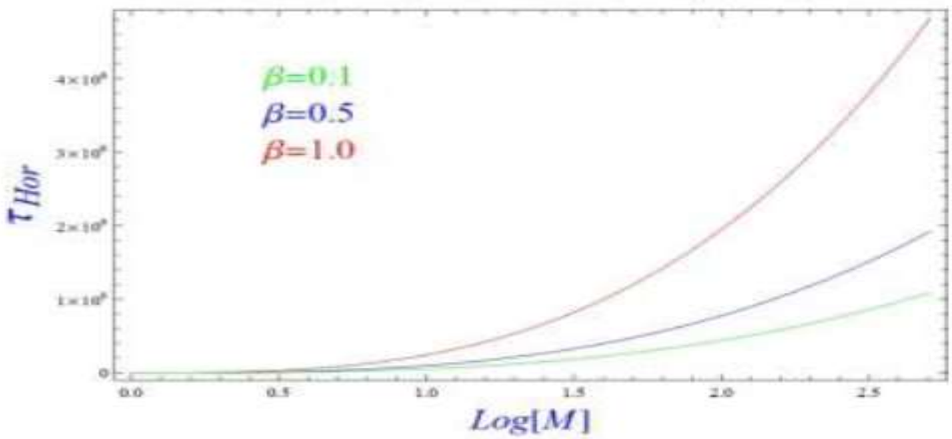

Figure 2. The dependence of the lifetime of the Hořava-Lifshitz black hole on its mass, for different values of the parameter $\beta = 1/\omega M^2$.

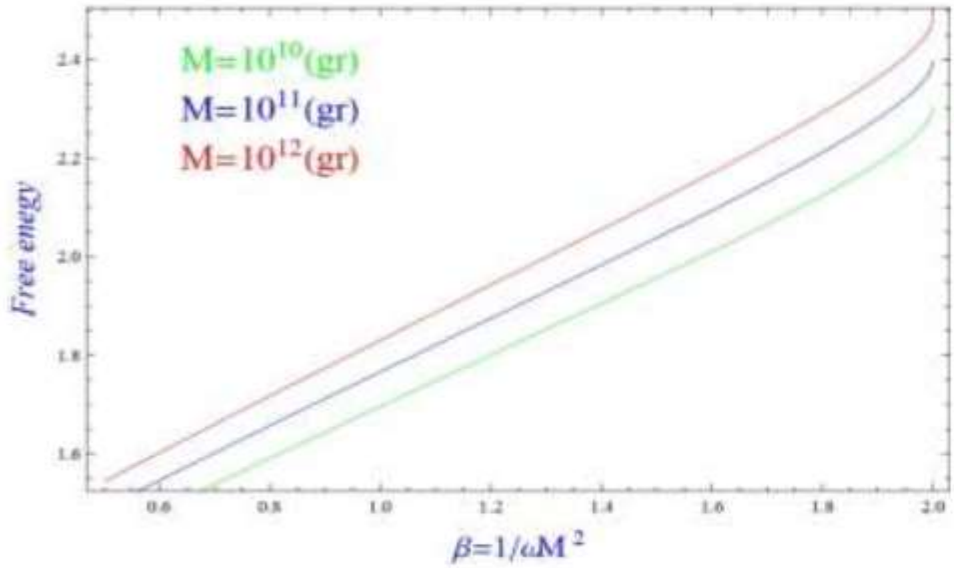

Figure 3. Dependence of free energy of the Hořava-Lifshitz black hole on its $\beta = 1/\omega M^2$ parameter, for different values of the mass.

Absence of $\beta = 1/\omega M^2$ parameter one can obtain $F = M/2$ that is free energy in Schwarzschild spacetime. Figure 3 shows dependence of the free energy from the $\beta = 1/\omega M^2$ parameter.

Now we consider the specific heat capacity of the Horava-Lifshitz black hole.

WEB OF SCIENTIST: INTERNATIONAL SCIENTIFIC RESEARCH JOURNAL
ISSN: 2776-0979, Volume 3, Issue 1, Jan., 2022

Considering the following $dQ = c^2 dM$ and in a system of geometrical units $c^2 = G = 1$

$$C = \frac{dQ}{MdT} = \frac{dM}{MdT} = -2\pi M \frac{\beta}{\Lambda}\left[1+\left(1+\frac{8}{\beta}\right)\left(\sqrt{1+32\frac{\Lambda}{\beta^2}+\frac{8}{\beta}\sqrt{1-\frac{\beta}{2}}}\right)^{-1}\right]^{-1}, \qquad (18)$$

Figure 4 shows dependence of the specific heat capacity of Hořava-Lifshitz black hole from the $\beta = 1/\omega M^2$ parameter.

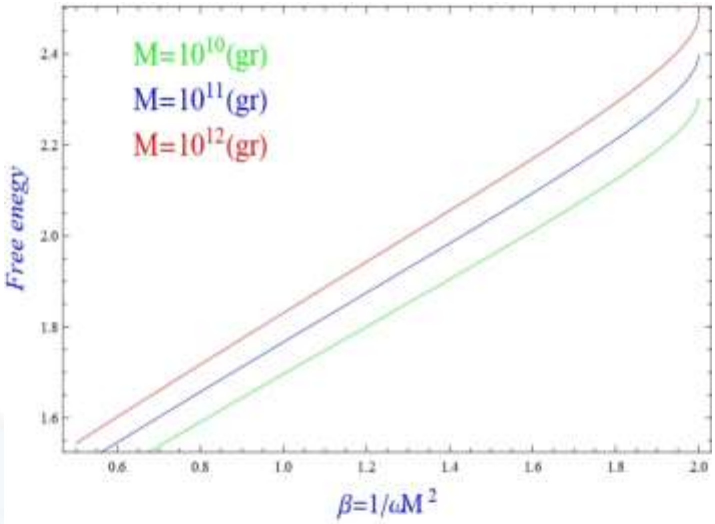

Figure 4. Dependence of the module specific heat capacity of the Hořava-Lifshitz black hole on its mass, for different values of the $\beta = 1/\omega M^2$ parameter

Conclusions

In this paper thermodynamic properties of the black hole, in the Hořava-Lifshitz gravity model in particular the Hawking radiation and evaporation of the Hořava-Lifshitz black hole have been studied. Our main results can be summarized as follows: We have studied the thermodynamic properties of black hole in the Hořava-Lifshitz gravity model. In the presence of the free parameter that regulates the ultraviolet behavior of the theory, an exact analytical expression for the thermodynamic quantities such as the temperature, entropy, heat capacity and free energy have been found.

We have also studied the Hawking radiation and the life time of Hořava-Lifshitz black hole. It is argued that at small values of the () parameter, the Hořava-Lifshitz black hole temperature is increased, hawking thermal radiation process is accelerated and its lifetime is shortened.

Acknowledgements

We would like to express our gratitude to Bobur Turimov, Ahmadjon Abdujabbarov and Bobur Toshmatov for their advice in the formation of this article.

References

1. Rovelli, C. (2004) Quantum Gravity. (Cambridge University Press)
2. Hořava, P. (2009) Quantum gravity at a Lifshitz point. Phys.Rev.D. 79, 084008.
3. Jiayi Hu, Jiliang Jing. (2018) Dirac quasinomal modes and area spectrum of Hořava-Lifshitz Black holes. Annals of physics. 389, 136-147.
4. Hawking, S.W. (1974) Particle creation by black holes. Commun. Math. Phys. 43, 199-220.
5. Cai, R.G and Wang, A. (2010) Singularities in Hořava-Lifshitz theory. arXiv:1001. 0155.v2[hep-th]
6. Cai, R.G, Cao, L.M and Ohta, N. (2009) Toplogical black holes in Horava-Lifshitz gravity. Phys.Rev. D 80 024003
7. Cai, R.G, Li, Y and Sun, Y.W. (2009) On the z=4 Horava-Lifshitz gravity. Journal of high energy physics, 06. 010.
8. Sadeghi J, Setare M.R and Pourhassan B. (2008) Two dimensional black hole entropy, Eur. Phys. J. C 53. 95.
9. Cadoni M, Carta P. (2001) 2D black holes, conformal vacua and CFTs on the cylinder. Phys. Lett. B 522. 126.
10. Sadeghi J., Setare M.R and Pourhassan B. (2009) Entropy of extremal black holes in two dimensions. Acta Phys. Pol. B 40. 251
11. Sadeghi J., Pourhassan B. (2009) Entropy function in the Liouville theory. Int. J.Theor. Phys. 48. 3526.
12. Sadeghi J., Pourhassan B. and Pourasadollah F. (2013) Thermodynamics of Schrodinger black holes with hyperscaling violation. Phys. Lett. B 720. 244.
13. Sadeghi J., Pourhassan B. and Asadi A. (2014) Thermodynamics of sting black hole with hyperscaling violation. Eur. Phys. J. C 74. 2680.
14. Hendi S.H., Panahiyan S., Upadhyay S. and Panah B.E. (2017) Charged BTZ black holes in the context of massive fravity's rainbow, Phys. Rew. D 95 084036.
15. Park M.I. (2009) The black hole and cosmological solutions in IR modified Horava-Lifshitz gravity. arXiv:0905.4480v5 [hep-th]
16. S.W. Hawking. (1977) Scientific American 236. 34